

ANNUITIES *7th ed.*
UPON
LIVES:

OR,

The VALUATION of
ANNUITIES upon any
Number of LIVES; as
also, of REVERSIONS.

To which is added,
An APPENDIX concerning the
EXPECTATIONS of LIFE, and
Probabilities of SURVIVORSHIP.

By A. DE MOIVRE. F. R. S. *K.*

L O N D O N,

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ANNUITIES
FROM
FIVE



6 Ba



TO
THE MOST HONOURABLE
THOMAS
EARL OF MACCLESFIELD,
Lord High Chancellour
OF
Great Britain.

MY LORD,



Should not presume to
inscribe the following
Piece to your Lordship,
were it not that the Subject
it treats of has been made the
Entertainment of some of Your
leisure Hours.

As

2 DEDICATION.

As this Essay is founded partly on Mathematical Reasoning, and partly on those Probable Conjectures that may be derived from Fact and Experience ; I thought I could not submit to a better Judge than Your Lordship, the Truth of this Theory, the Consequences that flow from it, and their Agreeableness to the Rules of that Equity which ought to preside in Contracts.

I am encouraged to hope for a favourable reception of this Performance, both from Your Lordship's known Goodness, and your noble Inclination to promote Arts and Sciences, of which Your Lordship

DEDICATION. 3

ship has upon all Occasions
shewn Your self the Great
and Glorious Patron.

THESE Motives, which of
themselves were more than
sufficient to induce me to give
Your Lordship this Testimo-
ny of my humble Respects,
are also heightened from the
Consideration of those Excel-
lent Qualities which so Emi-
nently distinguish Your Lord-
ship, and those Singular Virtues
of which the Influence is so
universally Beneficial.

I shall no longer detain
Your Lordship, but only re-
quest that You would be plea-
sed to look upon the Liberty
of this Address, as an effect
of

4 DEDICATION.

of that high Veneration which
I have constantly professed for
Your Lordship, and of the
earnest desire I ever had of
declaring my self,

MY LORD,

Your Lordship's

most Obedient, and

most Devoted

Humble Servant,

Jan. 1. 172 $\frac{4}{5}$.

A. DE MOIVRE.



P R E F A C E.



THE Method of Calculating the Values of Annuities upon Lives, seems never to have been perfectly understood 'till Dr. HALLEY first published the Rules of it; which may be seen in the Philosophical Trans. N^o. 196.

ALTHOUGH there is some Ground to believe that what he writ upon that Subject was partly occasional; yet there appears in it the same exquisite Judgment and Sagacity that has always shewn it self in all his other Productions.

AS this Calculation chiefly depends on the various Degrees of Probability which Lives of different Ages have to continue in Being, so it was previously necessary that a Table should be constructed which might at one View exhibit the State of Life in Respect to its probable Duration; Such a Table was accordingly constructed in the best Form that could be contrived, out of five Years Observations upon the Bills of Mortality of Breslaw the Capital of Silesia.

BUT notwithstanding the Pains taken by that excellent Mathematician in fitting a true Algebraic Calculation

A

culatation to his *Table of Observations*, yet he was sensible that both his *Table and Calculations*, were capable of farther *Improvements*; of this he expressed his Sense in the following Words: Were this *Calculus* founded on the Experience of a very great Number of Years, it would very well be worth the while to think of Methods to facilitate the Computation of Two, Three or more *Lives*.

FROM whence it appears that the *Table of Observations*, being only the Result of a few Years Experience, is not so entirely to be depended upon as to make it the Foundation of a fixt and unalterable *Valuation of Annuities for Life*, and that even admitting such a *Table* could be obtained as might be grounded on the Experience of a very great Number of Years, still the Method of applying it to the *Valuation of several Lives* would be extreamly laborious, considering the vast Number of Operations that would be requisite to combine every Year of each *Life* with every Year of all the other *Lives*.

I own, that in the Method which I have taken to make the Calculation of *Lives* easy, I have not made use of any other *Table* than of that which has been constructed by Dr. HALLEY; for which Reason, I am contented it should be thought that the *Values of Annuities*, as deduced from my Calculation, are still liable to some Exceptions, and need not therefore prejudice any established Practice of Estimation.

WHEREFORE all I can in Justice pretend to, is to have contrived such Rules as will make the *Valuation of Lives* easy, whenever it shall so happen that a *Table* is produced that can be depended upon, as to the Sufficiency of the Number of Observations on which it is built, the Accuracy of the Observers, and the Skill of

of those who construct the Table from the Observations.

I have been very particular in all the Questions that can be formed about Lives, and have rather chosen to insert in this Tract some few Problems that do not so immediately relate to Practice, than omit any Thing which might be thought essential to my Subject.

AFTER I have shewn how to estimate the Value of One Life, I proceed to the Consideration of Two, Three or as many Lives as may be assigned, and determine by an easy Calculation, 1st, The Values of joint Lives; that is, the Value of an Annuity which is to continue so long as the Lives concerned are all of them subsisting together; 2^{dly}, The Value of the longest Life, that is, the Value of an Annuity which is to continue so long as any One of the given Lives are in Being; 3^{dly}, The Value of all Sorts of Reversions, whether of One Life after One, of One after Two, of Two after One, &c. whether those Reversions be for Lives nominated at the Time of the first Possession, or to be nominated when the first Possession expires.

WHAT is chiefly to be regarded in the Rules above mentioned, is the Justness of their Conclusions; for admitting that several Lives singly taken are rated at their proper Values; then the Value of an Annuity upon all those joint Lives, or upon the longest of them, is determined by necessary Consequence; thus if Two Lives singly taken, be each of them worth 10 Years Purchase, and the Rate of Interest be 5 per Cent, then it will follow that an Annuity upon the longest of them will be worth 13.44; and in the same Manner, if Three Lives singly taken, be each of them worth 10 Years Purchase; then an Annuity upon the longest of them will be worth 15.14.

TO the End, that the Rules by me delivered may be of use to all Sorts of Readers, they have been expressed in Words at length, and the greatest Part of them illustrated by proper Examples ; as for the Demonstration of those Rules, it will easily be apprehended by those who being conversant in Algebra, are also acquainted with the First Rudiments of the Doctrine of Chances.

AFTER I have proceeded thus far, I subjoin an Appendix that has a very natural Connexion with the Things before demonstrated ; it contains a Method whereby taking still the Table of Observations to be our Guide, the Expectations of Life, and Probabilities of Survivorship are easily determined.

THE Expectation of Life, is that Duration which may justly be expected from a Life of a given Age ; and is properly a Medium between the longer and shorter Durations of a great Number of preceeding Lives, from the Time of their having attained the Age given, to the Time of their Extinction. With the present Value of this Expectation, the Proprietor of a Life may be said, in some Sense, to have purchased an Annuity for Life, of which the Rent is paid him in actual Duration, and thereby to have taken his Chance of an uncertain Duration, as an Equivalent for that fixt Duration he is entitled to by the Right of his Expectation.

TO facilitate the Computations relating to that Subject, I resume an Hypothesis which I had begun to introduce in the first Part of this Tract : It consists in supposing that the Number of Lives existing of any Age is proportional to the Number of Years intercepted between the Age given, and the Extremity of Old Age ; thus if there be 36 Lives of the Age of 50, we may imagin that in the succeeding Years, there will remain respectively 35, 34,

33, 32, &c. 'till they be all extinct in the 86th Year, which is supposed to be the utmost Term of Life. This Hypothesis will appear in several Respects to have some Foundation in Nature; 1st, As it assigns a Term to human Life, which is very rarely exceeded; 2dly, As it makes Life more and more obnoxious to decay; 3dly, As it has a near Conformity with the Table of Observations. After this Hypothesis has been established, I find Means to correct its Conclusions by help of the Table it self, and to bring them to so near a Degree of Coincidence with those deduced Year by Year from the Table, that it may be a doubt whether the Difference arising from the Comparison be not a Balance for some Unaccuracy that will still remain in the best Observations.

THE Conclusions derived immediately from my Hypothesis have a very great degree of Simplicity, of which to give some Instances; suppose 1° it be required to find the Expectation of a Life of which the Age is given; the Rule for finding it, is expressed in these few Words; take the Half Difference between the Age given, and the Exremity of Old Age supposed at 86, and that half Difference is the Thing required; thus if the Age be 50, then the Difference between 50 and 86 being 36, whereof the Half is 18; it may be concluded that 18 Years is the Duration which a Life of 50 may justly expect; but if the End of that Interval once happens to be attained by the Life in Question, then it acquires a farther Expectation of 9 Years, and so on: 2° let it be required to find the joint Expectation of Two equal Lives; the Rule will be to take one third of the Difference between their common Age and 86; thus if Two Lives be each of them 50, it may be expected that both of them together may continue in Being during an Interval of 12 Years; and if the Number of equal Lives be Three, Four, or Five, &c. then the Time due to their joint Continuance is respectively expressible by $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c.

&c. of the Difference between their common Age and 86;
 3° If there be ever so many equal Lives, then the Time due to the longest of them, that is the Time due to that Life which survives all the rest, may also be determined; for if there be Two, Three or Four equal Lives, then $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. of the Difference between their common Age and 86, will respectively express the Duration required.

I believe it will not be amiss, in this Place, to obviate a Difficulty that may be formed against some of the foregoing Conclusions; for it may be objected that admitting our Hypothesis to be true, then the Value of an Annuity for an Age of 50, would seem to be equivalent to an Annuity certain for the Term of 18 Years; now it appears from the 1st Problem, wherein this Hypothesis has been assumed, that an Annuity for an Age of 50, is worth only 10.34 Years Purchase, which being the present Value of an Annuity certain for the Term of 15 Years or thereabouts; it might from thence be infer'd that the same Principle leads to Two contradictory Conclusions.

IN Answer to this, I desire it may be considered that the Value of an Annuity is partly regulated by the Interest which the Purchaser makes of his Money, and partly by the Chance of Life; that the fixing the Value of 10.34 Years Purchase to a Life of 50 is accidental, being derived from an implied Supposition, that the Interest allowed to the Purchaser is at the Rate of 5 per Cent. that, as the Interest sinks, the Value of the Annuity rises and vice versa; and that in the Estimations made of the Expectations of Life, there is no Regard to be had to the Rate of Interest, the Years remote from the Age given, being as valuable in Point of Duration as those that are next to it, and consequently that no just Estimation can be made of the Expectation

ation of Life, but as it is compared with an Annuity of which the Purchase admits of no Discount, and to which there is a Payment due, even to the last Moment of Life.

IT may perhaps seem somewhat strange that the Speculation concerning Survivorship should be postponed to the Rules given for Settling the Values of Reversions; when those Values seem to suppose that the Chance of Survivorship is already known.

BUT it will not be difficult to account for the Steps taken in this Matter, for although the Notion of Survivorship may successfully be employed in determining the Values of Reversions, yet it will be found that the common Cases belonging to that Subject, are so easily derived from a proper Combination of single and of joint Lives, that any other Consideration would have been entirely superfluous.

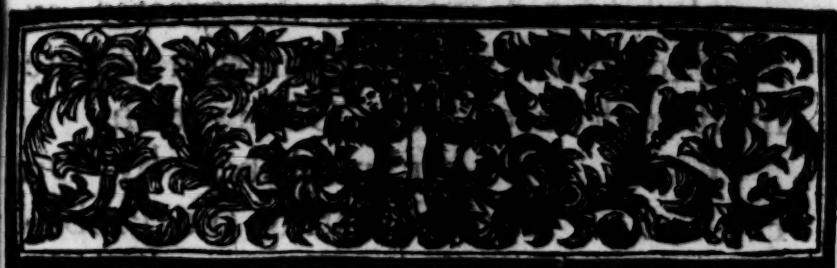
NEVERTHELESS, the Probability of Survivorship may be useful in resolving some extraordinary Cases relating to Reversions, viz. when it so happens that the Right of Reversion does not necessarily descend to the Life in Expectation, after the previous Demise of the Life in Possession; for a Life may intervene between the present Possessor and the Expectant, to which the Right of Reversion is immediately devolved, upon its surviving the Life in Possession, by Means whereof the Expectant is for ever excluded the Right of Reversion, although he should survive the Person interposed between him and the present Possessor; in this and other Cases of the like Nature, the Chance of Survivorship regulates the Value of the Expectation, of which it has been thought sufficient to give one single Instance, which may be seen in my XXVII. Problem.

THE

THE Rules delivered in this Tract requiring all along that the Extent of Life should be considered as confined, I have thought it convenient to fix the utmost Limit of it in the 86th Year, whereby I have nearly conformed my self to the Table of Observations ; but as some exceptionable Consequences might perhaps flow from too rigid an Application of that Principle to those Years of Life that border upon its Extremity ; it will not be improper to declare, that my Meaning, in fixing that Term, imports no more than this, viz. that the Time of contracting for Annuities being commonly very remote from the 86th Year of the Life purchased, it is not likely that any Consideration will then be given for the Chance of receiving the Rent of that Year, which will produce the very same Conclusions in Theory, as if the Extremity of that Year were never attainable.

Page 13. Line 13. for 10.15 read 10.34
Page 56. last Line. for B. read C.





OF THE
VALUATION
OF
ANNUITIES
FOR
LIFE.



N estimating the Values of ANNUITIES upon LIVES, Regard must be had to the Interest which Money bears, and to the Probability of the Lives continuing a longer or shorter Time.

B

THE

THE Rate of Interest is generally regulated by Law, but the greater or less Probability of the Duration of Life must be deduced from Observation.

THE TABLE published by Dr. HALLEY for estimating the Probabilities of Life being very artificially contrived, and grounded on such Observations, as one may reasonably suppose have been made with all the Exactness possible, we have annex'd it to this Treatise, in order to make use of it as Occasion shall require.

IT exhibits at one View the Probability that a Person of any Age proposed, may live any given Number of Years.

IT is distinguished into several Columns, shewing alternately the Age, and the Number of Persons living of that Age, both which are always written over-against one another.

SUPPOSE that by the help of this TABLE we would know what the respective Probabilities are for a Man of Thirty, to live 1, 2, 3, 4, 5, &c. Years.

LOOK

LOOK for the Number 30, in one, of the *Columns* of Age, under which Number you will find 31, 32, 33, 34, &c. over-against the said Number 30 in the next adjacent *Column* on the Right Hand, you find 531, under which are written 523, 515, 507, 499, &c. each respectively corresponding in order to the Numbers written in the *Column* of Age; the meaning of which is, that out of 531 Persons living of the Age of 30, there remain but 523, 515, 507, 499, &c. that attain to the respective Ages of 31, 32, 33, 34, &c. and who consequently do from that Term of 30, live 1, 2, 3, 4, &c. Years respectively.

NOW if out of the 531 Persons living of the Age of 30, there remain but 523 after one Year expired, it follows that the Number of Persons dead in that Time is the Difference between 531 and 523, viz. 8, and consequently the Number of Chances that a Person of that Age has to live one Year, is to the Number of Chances he has to die within that Year, as 523 to 8.

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BUT if we make a *Fraction* whose *Numerator* shall be the Number of Chances for living, and the *Denominator* the Number of Chances both for living and dying, that *Fraction* will be the Measure of the Probability of Life's continuance.

WHEREFORE, if the Numbers 523, 515, 507, 499, &c. answering to the respective Ages, 31, 32, 33, 34, &c. be written as *Numerators* of so many *Fractions*, whereof the common *Denominator* be 531, answering to the Number 30, which is the Term from which we begin to reckon; these *Fractions* $\frac{523}{531}$, $\frac{515}{531}$, $\frac{507}{531}$, $\frac{499}{531}$, &c. will express the Probabilities of a Man's of 30, living to the respective Ages of 31, 32, 33, 34, &c. that is of living 1, 2, 3, 4, &c. Years computed from the Term 30.

THIS being admitted, suppose it be required to find the Value of an ANNUITY for LIFE, for a Person of 30 Years of Age.

LET the ANNUITY be 100 l. and the Rate of Interest 5 per Cent.

SINCE

SINCE the present Value of an ANNUITY of 100 *l.* to be received a Year hence (Discount being made at 5 *per Cent.*) is 95.23 it follows that such would be the Value of the first Year's Rent, were the Purchaser sure to live one Year ; but his living one Year, being only probable according to that Degree of Probability which is measured by the *Fraction* $\frac{523}{531}$, we must conclude that the foregoing Sum of 95.23 ought to be diminished in the Proportion of 523 to 531, or, in other Words, that it ought to be multiplied by the *Fraction* $\frac{523}{531}$, which will reduce it to 93.80, and this will be the Value of the first Year's Rent.

IN the same manner may the second Year's Rent be calculated ; for were it certain that this Rent would be received, the present Value of it would be found to be 90.70 ; now its being received is only probable according to that Degree of Probability which is measured by the *Fraction* $\frac{512}{531}$; wherefore the Sum 90.70 being multiplied by the said *Fraction*, the Product will be 87.90,

87.90, which will be the present Value of the second Year's Rent.

AND by the like Process, the present Values of the 3^d, 4th, 5th, &c. Years Rents to the utmost extent of Life, may be calculated, and all these several Values being added together, the Sum will be the present Value of an ANNUITY upon a Life of Thirty, which will be found to be about 1300*l*.

ALTHOUGH the Number of *Operations* that are to be made in order to estimate one single LIFE be very numerous, yet that being once determined, the Value of the next younger Life may easily be obtained.

THUS suppose it were required to determine the Value of a Life of Nine and Twenty, from the given Value of a Life of Thirty, we may argue thus,

THE present Value of the first Year's Rent, upon a Certainty of its being received, would be 95.23; but its being received is only probable according to that Degree of Probability which is measured by the *Fraction* $\frac{531}{539}$, wherefore the present Value of
that

that Rent upon the Contingency of the Life's failing in the first Year, is the Product of 95.23 by $\frac{531}{539}$, that is 93.82.

LIKEWISE, the present Values of all the succeeding Rents will be 1300, when once the Age of 30 is attained, as we have seen before.

BUT it being uncertain whither that Age will be attained to, that Value ought to be diminished in the Proportion of 531 to 539, which will reduce it to 1280.7, and this Sum the Purchaser might engage should be paid after the Expiration of one Year whether he lived or not, for the Expectation of enjoying the ANNUITY after the Age of Thirty.

BUT if the Sum of 1280.7, which is payable after one Year, be discounted at the Rate of 5 per Cent. its present Value will be reduced to 1219.7, which being added with the present Value of the first Year's Rent, viz. 93.82, the Sum will be 1313.52, which will be the Value of an ANNUITY for a Life of Nine and Twenty.

By

By help of the Method hitherto explained, those that will take the Pains may, if they think fit, compose TABLES of the several Values of those ANNUITIES for any Age proposed, and for any Rate of Interest that shall be fixed upon.

BUT till that be done, it will be convenient to try whether there be not some easy practical Method whereby the Values of those ANNUITIES may be determined very near as accurately as if they had been deduced Year by Year from the *Tables of Observation*.

THIS may be so much the more readily undertaken, that any small Difference that may arise from the two Methods of Calculation is not to be regarded in a Subject of this Nature.

IN order to it, we may consider that whatever be that Law which is observed by Nature in the perpetual Decrements of human Life, that Law must, conformably to all the other Laws of Nature, be such as to proceed regularly, at least for some short Intervals of Time.

WHERE-

WHEREFORE if we examin Dr. HALLEY'S Tables with a View of discovering what Regularity there may be in the perpetual Decrement of Lives, and how far that Regularity is extended, we shall find that out of 646 Persons of twelve Years of Age, there remain 640 after one Year, 634 after two, 628, 622, 616, 610, 604, 598, 592, 586, after 3, 4, 5, 6, 7, 8, 9, 10 Years respectively.

FROM whence we may conclude, that for a Space of ten Years, there is a Decrement of the Probabilities of Life, according to the Order of an Arithmetic Progression, these Probabilities being $\frac{634}{640}$, $\frac{628}{640}$, $\frac{622}{640}$, &c.

IT is carefully to be observed that the Decrement of these Probabilities is here considered as beginning from the same Term, and that as the first *Fraction* above written, represents the Probability of living from Twelve to Thirteen, so the next represents the Probability of living from Twelve to Fourteen, the next to this, the Probability of living from Twelve to Fifteen, &c.

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IN the same Manner, if we take the Term 42, we shall find that for 8 Years together, the Probabilities of living to 42, 43, 44, 45, &c. are respectively represented by the *Fractions* $\frac{417}{427}$, $\frac{407}{427}$, $\frac{397}{427}$, $\frac{387}{427}$, &c. which decrease in *Arithmetic Progression*.

AND in taking greater *Intervals*, we find that out of 531 Persons of Thirty Years of Age, there are but 262 remaining after Twenty Eight Years; and again that all those who are then remaining are very near extinct after Twenty Eight Years more, *viz.* in the 86th Year, which we may well suppose without any considerable Error to be the utmost Term of human Life.

NOW since those that are living of Thirty Years of Age are reduced to about one Half in Twenty Eight Years, and that in Twenty Eight Years more, the remaining Half is very near extinct; we may suppose that, during the Interval of Fifty Six Years, the Decrements of Life considered from the Term of Thirty, are constantly equal.

LET

for LIFE.

II

LET us therefore consider, 1^o what would be the result of an *Hypothesis* that makes the Probabilities of Life to decrease in *Arithmetic Progression* : 2^o how far the Calculations deduced from it agree with the Tables : 3^o what Corrections are necessary to be made to it where it varies from the Tables.

PROBLEM I.

SUPPOSING the Probabilities of Life to decrease in *Arithmetic Progression*, to find the Value of an *Annuity upon such a Life*.

SOLUTION.

LET 1 , be the Rent ; r , the Rate of Interest, that is the Interest of one Pound joined with its Principal ; n , the Number of Years comprehended between the Term given, and the last Term in which the Probability of Life is supposed to be extinguished ; Let also P be the present Value of an *Annuity* certain for the given Number of Years,

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THIS being supposed, the Value of an *Annuity* upon such a Life will be $1 - \frac{r}{r-1}P$, which may be thus expressed in Words at Length.

TAKE the Value of an *Annuity* certain, for so many Years as are intercepted between the Age given, and the utmost Term of Life supposed to be at Eighty Six compleat; multiply this Value by the *Rate of Interest*, and divide the Product by the Number of Years belonging to the *Annuity*; let the Quotient be subtracted from Unity, and let the Remainder be divided by the Interest of one Pound: This last Quotient will express the Value of an *Annuity* for the Life given.

THUS, suppose it were required to find the present Value of an *Annuity* of one Pound for a Life of Fifty, Interest being at 5 per Cent.

THE Number of Years intercepted between Fifty and Eighty Six, is Thirty Six; therefore let the Value of an *Annuity* certain for Thirty Six Years be taken out of the *Tables of*
Annuity

Annuities; this Value will be found to be 16.5468.

LET this be multiplied by 1.05 *Rate of Interest*, the Product will be 17.3741.

LET this Product be divided by the Number of Years, viz. by 36; the Quotient will be 0.4826.

SUBTRACT this Quotient from 1, the Remainder will be 0.5174.

LASTLY, divide this Quotient by the Interest of one Pound, viz. by 0.05, and the new Quotient, viz. 10.15 will be the Value of an *Annuity* of one Pound for a Life of Fifty.

DEMONSTRATION.

SINCE, by Hypothesis, the Probabilities of Life decrease in *Arithmetic Progression*, it follows that they may be represented by the following Series,

$\frac{n-1}{n}, \frac{n-2}{n}, \frac{n-3}{n}, \frac{n-4}{n} \dots \frac{n-n}{n}$,
whose Terms being severally divided by the Terms of the *Geometric Progression*, $r, rr, r^3, r^4, \&c.$ the new Series
re-

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resulting from these Divisions, viz.

$$\frac{n-1}{nr}, \frac{n-2}{nr^2}, \frac{n-3}{nr^3}, \frac{n-4}{nr^4}, \dots \frac{n-n}{nr^n}$$

will express the Value of all the Rents to be received yearly, and estimated in present Money; and consequently the Sum of the Terms of this Series is the present Value of the *Annuity* required.

BUT in order to find the Sum of all these Terms, it is to be observed that the Series which they compose may be divided into two other Series, viz.

$$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} \dots \dots \dots + \frac{1}{r^n}$$

$$\frac{1}{nr} + \frac{2}{nr^2} + \frac{3}{nr^3} + \frac{4}{nr^4} \dots \dots \dots + \frac{n}{nr^n}$$

whereof the Second is to be subtracted from the First.

THE Sum of the first Series may be look'd upon as given, being an *Annuity* certain for a Number of Years expressed by n , and may either be derived from Tables exhibiting the Values of *Annuities* for any Term of Years, and a given Interest, or be readily calculated by the known Method of summing up the Terms of a *Geometric Progression*; this Sum being contained in

In the short Expression $\frac{1 - \frac{1}{r^n}}{r - 1}$, which

we may suppose equal to P .

THE Method of calculating the Sum of the second Series is not so vulgarly known, but it may be derived from a general *Theorem* in the *Doctrine of Chances*, pag. 132, which shews

it to be $P + \frac{\frac{r}{n}P - 1}{r - 1}$. Wherefore, we

may conclude that the present Value of the *Annuity* required is $\frac{1 - \frac{r}{n}P}{r - 1}$.

IN order to compare this, with the Value deduced from Dr. HALLEY'S Tables, Let us suppose first, the Age for which the *Annuity* is to be calculated to be Thirty ; in this Case, $n = 56$, let the *Rate of Interest* be $= 1.06 = r$; then P will be $= 15.524$; from which *Suppositions*, the Value of the *Annuity* required will be found to be 11.61, which differs from that given by Dr. HALLEY, viz. 11.72, by very little more than $\frac{1}{10}$ of a Year's Purchase.

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LET us now suppose the Age required to be Forty, then the Value required will be found to be 10.70, which exceeds that given by Dr. HALLEY, viz. 10.57 only by the small *Fraction* 0.13.

IF the Age be Fifty, then the Value required will be found to be 9.49, which differs in Excess from the Values of the Tables by 0.28.

IF the Age be Sixty, the difference in excess will be 0.23.

IF the Age be Seventy, the difference in excess will be 0.18.

FROM whence we may conclude, that let the *Rate of Interest* be what it will, the Value of an *Annuity* for an Age not less than Thirty, will by help of our Canon, be found nearly the same as the Values deduced from the Tables.

BUT if we try by our Canon what will be the Value of an *Annuity* for an Age less than Thirty, suppose Twenty, we shall find that this Value will be 12.30; whereas the Value found by the Tables is 12.78; therefore the dif-

difference .058 now lies on the contrary Side of what it did after Twenty.

AND if we try what will be the Value of an *Annuity* for an Age of Ten, we shall find this Value to be 12.84, which is deficient from the Value found by the Tables, viz. 13.44 by 0.60.

LASTLY, if we calculate what would be the Value of an *Annuity* for an Infant One Year old, we shall find this Value to be 12.57, which differs in Excess from the Value found by the Tables, viz. 10.28 by 2.29.

The Reason of which Variations may easily be assigned; for the Decrements of Life towards old Age are something quicker in the Tables, than the Decrements supposed in our Hypothesis, for which Reason the Values of *Annuities* deduced from the Tables are a little below those that are deduced from our Canon.

BUT towards the Beginning of Life, viz. from the Age of Ten and upwards, the Derements of the Tables are

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some-

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something slower than in our Hypothesis; for Instance, if we examin the Numbers set down in the Tables over-against 20 and 21 we find 598, 592, and consequently the first Decrement after the Age of Twenty, being 6 in 598, it follows that if the succeeding Decrements were to continue in that Uniformity, a Life of Twenty could not be supposed to be necessarily extinguished till 100 Years after, which might make it reach to 120, and this Difference between the Decrements of the Tables, and those of our Hypothesis, are the Occasion of the second Variation observed in the *Estimation of Annuities*.

THE third Variation, which is the most considerable, is occasioned by the Increments of *Life*, from Year to Year, between the first Year and the Twelfth; for according to the Tables, the Probabilities of living One, Two, Three, Four, &c. Years, all to begin from the First, are respectively $\frac{855}{1000}$, $\frac{798}{1000}$, $\frac{760}{1000}$, $\frac{732}{1000}$, &c. from whence it follows, that the Probability of living from the Second

cond to the Third, is greater than the Probability of living from the First to the Second, and that the Probability of living from the Third to the Fourth, is greater than the Probability of living from the Second to the Third, and so on; these Probabilities written in their natural Order being expressible by the *Fractions* $\frac{855}{1000}$, $\frac{798}{855}$, $\frac{760}{798}$, $\frac{732}{760}$, &c. of which every One is greater than that which preceds it, 'till we come to the greatest of all, viz. $\frac{640}{646}$, which expresses the Probability of living from the Twelfth to the Thirteenth Year, after which there is a perpetual Decrease to the End of Life.

NOW in the Hypothesis of a perpetual and equable Decrement of the Probabilities of Life, to begin from a Term given, there is also a perpetual, though not equable Decrement of the Probabilities of Life when estimated from Year to Year, which makes it that such an Hypothesis must differ the most from the Tables of Observations for the Twelve first Years of Life.

BUT although the Notion of an equable Decrement of Life to begin from the Age of Twelve, and to continue throughout the whole Extent of Life, do not exactly agree with the *Tables*; yet that Notion may successfully be employed in constructing a *Table* of the Values of *Annuities* for *Ages* not inferiour to Twelve, as will appear by the Consequences deduced from the following *Problem*.

PROBLEM II.

TO find the Value of an Annuity for a limited Interval of Time, during which the Decrements of Life may be considered as equal.

SOLUTION.

LET a and b represent the Number of People respectively living in the Beginning and End of the given Interval of Years, which Interval we will suppose $= n$; Let P be

be the Value of an *Annuity* certain for that Number of Years ; make $1 - \frac{r}{n} P = Q$, then will the Value of an *Annuity* granted for that Interval of Life be $= Q + \frac{b}{a} \times \overline{P} - Q$, which may be thus expressed.

1° Take by the preceding *Problem* the Value of an *Annuity* for a Life supposed necessarily terminated in the given Interval of Time.

2° Subtract this Value from the Value of an *Annuity* certain, to continue so many Years, as are contained in the given Interval.

3° Multiply the Remainder by the Number, which in the *Tables* terminates the given Interval, and divide the Product by the other Number, which answers to the Age given.

4° To this *Quotient* add the Value first found, and the *Quotient* will be the Value required.

THUS, suppose we would find the Value of an *Annuity* for an Age of Twelve, to continue Ten Years and no longer, and to cease from the
Time

Time that the Life may happen to fail within the Interval given : *Interest at 5 per Cent.*

THE Value of an *Annuity* for a Life, supposed to be necessarily extinguished in Ten Years is by the preceding *Problem*, 3.7843.

THIS being subtracted from the Value of an *Annuity* certain for Ten Years, viz. 7.7217, the *Remainder* will be 3.9374.

THIS *Remainder* being multiplied by the Number 628, which in the *Tables* answers to Twenty Two, terminating the given Interval, and divided by the Number 616, which in the *Tables* answers to Twelve, the Age given, the *Quotient* will be 3.8277.

THIS *Quotient* added with 3.7843, Value found by the first Article, the Sum will be 7.6120, which will be the Value of the *Annuity* required, and is just the same as if it had been deduced Year by Year from the *Tables*.

DEMONSTRATION.

SINCE the whole Interval between a and b is filled up with *Arithmetic Means*, it follows that the Number of People living in the Beginning and End of every Year of the Time given, will respectively be represented by the following Series, viz.

$$a, \frac{na - a + b}{n}, \frac{na - 2a + 2b}{n} \dots \dots b.$$

and consequently, the Probabilities of living 1, 2, 3, 4, &c. Years, to begin from the Term given, will be represented by the Series.

$$\frac{na - a + b}{na}, \frac{na - 2a + 2b}{na} \dots \dots \frac{b}{a}.$$

WHEREFORE the Value of an *Annuity*, granted for the limited Time comprehended within the given Interval of Years, will be expressed by the Series,

$$\frac{na - a + b}{nar} + \frac{na - 2a + 2b}{nar^2} \dots \dots \frac{1 + b}{ar^n}$$

which Series is divisible into two others, viz.

$$\frac{n-1}{nr} + \frac{n-2}{nrr} + \frac{n-3}{nr^2} \dots\dots\dots + \frac{n-n}{nr^n}$$

$$\frac{b}{a} \times \frac{1}{nr} + \frac{2}{nrr} + \frac{3}{nr^2} \dots\dots\dots + \frac{n}{nr^n}$$

BUT the Value of the first Series
 = Q by the first Problem, and the
 Value of the second is by the same

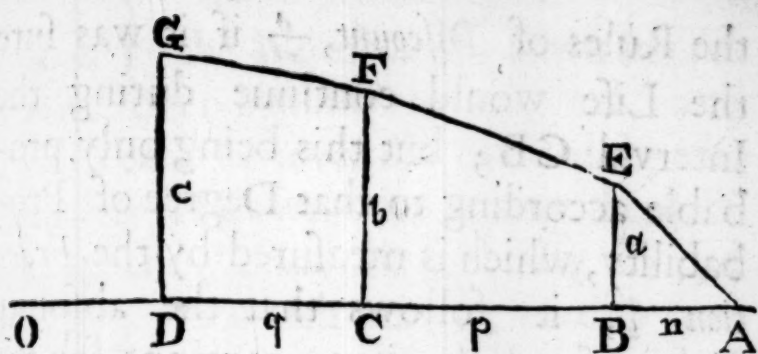
$$\frac{b}{a} \times \frac{P + \frac{r}{n}P - 1}{r-1} \text{ or } \frac{b}{a} \times \overline{P - Q};$$

Therefore the Value required is $Q +$
 $\frac{b}{a} \times \overline{P - Q}.$

COROLLARY.

SINCE the Decrements of Life
 may without any sensible Error
 be supposed equal, for any short In-
 terval of Time ; it follows that if
 the whole Extent of Life be divided
 into several shorter Intervals, whether
 equal or unequal ; the Values of *An-*
nuities for Life to be estimated from the
 Beginning of each of these Periods,
 may easily be calculated from the pre-
 ceding Theorem, conformably to any
Tables of Observations, and for any *Rate*
of Interest.

LET



LET OA represent the whole Interval of Life, intercepted between the first Instant of Birth at O , to the Extremity of old Age at A .

LET that *Interval* OA be divided into smaller Intervals of Time, AB , BC , CD , &c. respectively equal to n , p , q , &c. during the Continuance of which, the Decrements of Life considered from the respective Terms B , C , D , &c. may be supposed to be equal; Let BE , CF , DG , &c. respectively equal to a , b , c , &c. represent the Number of Persons living in the Beginning of each of these Periods of Time.

THE Value of an *Annuity* for the Interval AB is given by the first *Problem*; Let it be called A .

THIS same Value estimated from the Term C , would be, according to
 E the

the Rules of *Discount*, $\frac{A}{r^p}$ if it was sure the Life would continue during the Interval CB; but this being only probable according to that Degree of Probability, which is measured by the *Fraction* $\frac{a}{b}$, it follows that the absolute Value of an *Annuity* to continue for the Time AB, and to be estimated from the Term C, is $\frac{a}{b} \times \frac{A}{r^p}$; Now this Value being added to the Value of an *Annuity*, to continue during the Interval CB (which may be obtained by our second *Problem*) the Sum will be the Value of an *Annuity* for the Interval CA.

IN the same Manner, this last Value, which we may call B, being estimated from the Term C, will be reduced to $\frac{b}{c} \times \frac{B}{r^q}$; and this being added to the Value of an *Annuity* to continue during the Interval DC, the Sum will be the Value of an *Annuity* for the Interval DA.

AND by the same Method, the Value of *Annuities* may by Retrogression be found for any Age not inferior to Twelve.

AS for the first Twelve Years, the Process may be as follows. LET

LET the Number set down in the Tables, over-against the 2d, 7th, and 12th Year, be severally divided by the Number set down over-againg the first, and let the *Quotients* be severally divided by r , r^6 , r^{12} , which may be readily performed by *Logarithms*; Let the new *Quotients* be respectively called E , F , G , and let the Interval between the first and twelfth Year, viz. 11 be called q ; then will the Value of an *Annuity* granted for that Interval of *Life* which begins at 1, and terminates at 12, be very near equal to

$$\frac{q+1}{3q} \times \frac{1}{q+1} \times E + F + \frac{2q-2}{q+1} \times F.$$

UPON the Principles hitherto explained, I have calculated the Values of *Annuities* for every 10th Year for an Interest of 5 per Cent. which are as follows,

Age.	Yrs. Purch.	Age.	Yrs. Purch.
76	3.78	36	12.20
66	6.46	26	13.60
56	8.88	16	14.84
46	10.62	6	15.21
		1	11.70

*Of the Valuation of Annuities
upon several Lives.*

PROBLEM III.

SUPPOSING a Fictitious Life, of which the Number of Chances to continue yearly, be constantly equal to a , and the Number of Chances to fail, constantly equal to b ; so that the Odds of its continuing, during the Space of any one Year, be to its failing in that same Interval of Time as a to b ; to find the Value of an Annuity upon such a Life.

SOLUTION.

LET i be the Annuity, r the Rate of Interest; make $a + b = s$.

IT is plain from what has been shewn already, that the present Value of the first Year's Rent is $\frac{a}{r}$; of the Second $\frac{a}{r(1+r)}$, of the Third $\frac{a}{r(1+r)^2}$, &c. and so on; which Terms constituting a Geometric Progression, the Sum of it, is $\frac{a}{r}$.

THERE-

THEREFORE 1^o take the Number of Chances which the *Life* given has for its continuing one Year, and let that be made the *Numerator* of a *Fraction*.

2^o MULTIPLY the Number of all the Chances both for the *Life's* continuing and failing, by the *Rate of Interest*, and from the *Product* subtract the Number of Chances for its continuing, and let the *Remainder* be made the *Denominator* of the same *Fraction*.

3^o DIVIDE the *Numerator* by the *Denominator*, and the *Quotient* will express the Value of an *Annuity*.

THUS if there was a *Life* so constituted as that the Chances of its continuing any one Year, were to the Chances of its failing in that Year, constantly as 21 to 1, the Value of an *Annuity* upon such a *Life* would be worth exactly 10 Years Purchase, *Interest* being suppoed at 5 per Cent. as appears from the following Operation.

21 Number of Chances for continuing One Year, Numerator of the Fraction.

$\frac{1}{22}$ Chance for failing in that Year.
 $\frac{22}{21}$ Sum of all the Chances, which multiply'd
 by $\frac{1.05}{23.1}$ Rate of Interest.
 Product 23.1 from which subtract
 21 . Number of Chances for continuing
 Remain^r. 2.1 Denominator of the Fraction.
 Fraction $\frac{2.1}{23.1} = 10$ Years Purchase.

COROLLARY. I.

AN Annuity upon a fictitious Life being given, the Probability of its continuing any one Year is also given, for let the Value of such a Life be $= M$, then $\frac{a}{r + a} = M$, Therefore $\frac{a}{r} = \frac{Mr}{M + 1}$. that is

IF the Value of the Life be multiply'd by the Rate of Interest, and the Product be divided by the Value of the Life increased by Unity, the Quotient will express the Probability of the Life's continuing one Year, or thus

1^o MULTIPLY the Value of the Life by the Rate of Interest.

2^d FROM the Value of the *Life* increased by *Unity* subtract the preceding Product.

THEN the Proportion of the Product to the Remainder, will express the Proportion of the Chances which the *Life* has for continuing one Year, to the Chances it has for failing in that Year.

COROLLARY II.

IF a real *Life*, whose Value as deduced from the *Tables of Observations*, be worth 10 Years Purchase ; then such a *Life* is equivalent to a Fictitious *Life*, whose Chances for continuing one Year, are to the Chances of its failing in that Year, as 21 to 1.

COROLLARY III.

WHEREFORE, having calculated a *Life* from the *Tables of Observations*, and having obtained the true Value of it, we may transfer the Value of that *Life* to that of a Fictitious

tious *Life*, and find the Number of Chances it would have to continue or to fail yearly.

COROLLARY IV.

AND the Combination of two or more *real Lives*, will be very near the same as the Combination of so many corresponding *fictitious Lives*, and therefore an *Annuity* granted upon two or more *real Lives*, will be very near the same as an *Annuity* granted upon so many corresponding *fictitious Lives*, and the Values of the Reversions granted upon the *real Lives*, will be very near the same as those granted upon the *fictitious Lives*.

PROBLEM IV.

THE Values of two single *Lives* being given to find the Value of an *Annuity* granted for the Time of their Joint Continuance.

SOLU-

SOLUTION.

LET the Values be respectively M and P , r the Rate of Interest; then the Value of an Annuity upon the two joint Lives, will be

$$\frac{MPr}{M + 1 \times P + 1 - MPr}$$
; which may be thus expressed.

1° MULTIPLY together the Values of the Lives and the Product again by the Rate of Interest, and let the new Product be made the Numerator of a Fraction.

2° ADD Unity to each of the single Values, and let these two Values thus increased by Unity be multiplied together, and from the Product subtracting the Numerator before found, let the Remainder be made the Denominator of the said Fraction.

3° DIVIDE the Numerator by the Denominator, and the Quotient will be the Value of an Annuity to continue so long as both the Lives together are in being.

F

THUS

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THUS it will be found that if the two *Lives* singly taken, be respectively worth 8 and 10 Years Purchase, the Value of an *Annuity* upon the two joint *Lives* will be worth 5.6 Years Purchase, as will appear by the following *Operation*.

	8	Value of the first <i>Life</i> .
	10	Value of the second
Product	80	which multiplied by the
Rate of		
Interest	1.05	
	84.0	new Product, Numerator of
the Fraction.		
	9	first Value increased by 1
	11	second Value increased by 1
Product	99	
subtract	84.	Numerator first found
Remain ^r	15	Denominator of the Fraction.
Fraction	$\frac{84}{15}$	
	15)	84 (5.8 Value of the two
joint Lives.		

IN the same manner, it will be found that if the Values of the *single Lives* were respectively 10 and 12 Years Pur-

Purchase, the Value of an *Annuity* upon the two *joint Lives* would be 7.412 Years Purchase.

AND that if the Values upon the *single Lives* were respectively 8 and 12, the Values of an *Annuity* upon the two *joint Lives* would be 6.222 Years Purchase.

DEMONSTRATION.

LET x and y represent the respective Probabilities of the *Lives* continuing One Year, then the *Product* xy will express the Probability of their Joint continuance for One Year; and $xxyy$, the Probability of their Joint continuance for Two Years; and x^3y^3 the Probability of their Joint continuance for Three Years, and so on: Wherefore the Value of an *Annuity* granted for all the Time of their Joint continuance will be expressible by the following *Geometric Progression*, viz.

$$\frac{xy}{r} + \frac{xxyy}{rr} + \frac{x^3y^3}{r^3} + \frac{x^4}{r^4} \text{ \&c. whose}$$

Sum is $\frac{xy}{r - xy}$, but by the First Corollary

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lary

lary of the Third Problem, $x = \frac{Mr}{M+1}$,
 and by the same Corollary, $y = \frac{Pr}{P+1}$;
 Wherefore the Values of the two joint
Lives is $\frac{MPr}{M+1 \times P+1 - MPr}$ as had
 been determined.

PROBLEM V.

THE Values of two *single Lives*
 being given, to find the Value
 of an *Annuity* upon the *longest* of them;
 that is to continue so long as either of
 them is in being.

SOLUTION.

FROM the Sum of the Values of
 the *single Lives*, subtract the Va-
 lues of the two *joint Lives* found by
 the preceding Problem, the *Remainder*
 will be the Value of the *Annuity* requi-
 red.

THUS if the two *Lives* singly taken
 be respectively worth 8 and 10 Years
 Pur-

Purchase, the two joint *Lives* by the preceding *Problem* will be worth 5.6 ;
Therefore

From 18 Sum of the Values of the
single Lives
subtract 5.6 Value of the two joint
Lives.

remains 12.4 Value of an *Annuity* upon the longest of them.

DEMONSTRATION.

LET x and y be the respective Probabilities of the *Lives* continuing One Year ; then the Probability of their respective failing in that Year, are $1 - x$, and $1 - y$, which multiplied together, the *Product* viz. $1 - x - y + xy$ exhibits the Probability of their both failing in that Year, and this subtracted from Unity, the *Remainder*, viz. $x + y - xy$, expresses the Probability of one of them at least surviving the Year, and for the same Reason, the Terms $xx + yy - xxyy$ express the Probability of One of them at least surviving Two Years, and so on :
From

From whence arise the following *Geometric Progressions*, expressing the Value of an *Annuity* upon the *longest* of the two *Lives*, viz.

$$\begin{aligned} & \frac{x}{r} + \frac{xx}{rr} + \frac{x^3}{r^3} + \frac{x^4}{r^4} + \frac{x^5}{r^5} \quad \&c. \\ & + \frac{y}{r} + \frac{yy}{rr} + \frac{y^3}{r^3} + \frac{y^4}{r^4} + \frac{y^5}{r^5} \quad \&c. \\ & - \frac{xy}{r} - \frac{xxyy}{rr} - \frac{x^2y^2}{r^2} - \frac{x^3y^3}{r^3} - \frac{x^4y^4}{r^4} \quad \&c. \end{aligned}$$

of which the first is the Value of the first *Life* singly taken ; the second is the Value of the second *Life* singly taken ; and the third is the Value of an *Annuity* upon the two *joint* *Lives*, and therefore the Value of an *Annuity* upon the *longest* of two *Lives*, is the Sum of the Values of the two *Lives* lessened by the Values of the two *joint* *Lives*,

PROBLEM VI.

THE Values of three single *Lives* being given to find the Value of an *Annuity* granted for the Time of their *Joint Continuance*.

SOLU-

SOLUTION.

LET the single Values be respectively M , P , Q ; r the Rate of Interest; then the Value of an Annuity upon the three joint Lives will be

$$\frac{MPQrr}{M + 1 \times P + 1 \times Q + 1 - MPQrr}$$

which may be expressed as follows.

1° MULTIPLY the Values of all the single Lives together, and the Product again by the Square of the Rate of Interest, and let the Product be made the Numerator of a Fraction.

2° ADD Unity to each of the single Lives, and let all the Values thus increased by Unity be multiplied together; and from the Product subtracting the Numerator before found, let the Remainder be made the Denominator of the same Fraction.

3° DIVIDE the Numerator by the Denominator, and the Quotient will be the Value of an Annuity upon the three joint Lives.

THUS

THUS it will be found, that if the three Lives singly taken, be respectively worth 8, 10 and 12 Years Purchase, the Value of an *Annuity* upon the three *joint* Lives will be 4.630 as will appear by the following *Operation*.

	8	Value of the first <i>Life</i> .
	10	Value of the second.
	12	Value of the third.
	960	Product of the three
Values, which multiplied		
by	1.1025	Square of the Rate
of <i>Interest</i> .		
New Product	1058.4	Numerator of the
Fraction.		

		9	Value of the first <i>Life</i>
increased by 1.			
		11	Value of the second in-
creased by 1.			
		13	Value of the third in-
creased by 1.			
Product		1287	from which subtrac-
ing			
		1058.4	Numerator before
found.			

remains		228.6,	Demominator.
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Frac.

Fraction $\frac{1058.4}{228.6}$

228.6) 1058.4 (4.630 Value of the three joint Lives.

DEMONSTRATION.

LET x, y, z , represent the respective Probabilities of the *Lives* continuing one Year; then the *Product* xyz will express the Probability of their joint Continuance for one Year; and $xxyyzz$ the Probability of their joint Continuance for two Years; and $x^3y^3z^3$, the Probability of their joint Continuance for three Years, and so on. Wherefore the Value of an *Annuity* granted for the whole Time of their joint Continuance will be expressible by the following *Geometric Progression*, viz.

$$\frac{xyz}{r} + \frac{x^2y^2z^2}{r^2} + \frac{x^3y^3z^3}{r^3} + \frac{x^4y^4z^4}{r^4} \text{ \&c.}$$

the Sum of whose Terms is $\frac{xyz}{r - xyz}$, but by the First *Corollary* of the Third

Problem $x = \frac{Mr}{M+1}$, $y = \frac{Pr}{P+1}$, $z = \frac{Qr}{Q+1}$. Wherefore the Value of the three joint Lives is

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M P Q r r

$$\frac{MPQrr}{M + I \times P + I \times Q + I - MPQrr}$$

as had been determined.

PROBLEM VII.

THE Values of three single Lives being given to find the Value of an Annuity upon the longest of them.

SOLUTION.

FROM the Sum of the Values of the single Lives, subtract the Sum of the Values of all the joint Lives combined Two and Two; to the Remainder add the Value of the three joint Lives, the Sum will be the Value of an Annuity upon the longest of them.

THUS, if the three Lives singly taken be respectively worth 8, 10 and 12 Years Purchase, it will be found that the Value of an Annuity upon the longest of them will be worth 15.396 Years Purchase, as will appear by the following Operation.

for LIFE. 43

8 Value of the first *Life*.

10 Value of the second.

12 Value of the third.

30 Sum of all the single

Values.

5.6 Value of the first and second jointly taken, as may be deduced from the *Fourth Problem*.

6.222 Value of the first and third.

7.412 Value of the second and third.

19.234 Sum of all the Values combined Two and Two; which being subtracted from 30 Sum of all the single Values.

remains 10.766 to which adding

4.630 Value of the three joint *Lives*.

Sum 15.396 Value of the *Annuity* upon the longest *Life*.

DEMONSTRATION.

LET x, y, z represent the respective Probabilities of the *Lives* continuing one Year ; then the Probabilities of their respective failing in that Year are $1 - x, 1 - y, 1 - z$; which being multiplied together the *Product*, viz. $1 - x - y - z + xy + xz + yz - xyz$ will express the Probability of their all failing in that Year, and this subtracted from *Unity*, the *Remainder*, viz. $x + y + z - xy - xz - yz + xyz$ will express the Probability of their not all failing in that Year, ; otherwise, the Probability of one of the *Lives* at least surviving the Year, from whence, by following the *Step* of the Fifth *Problem*, may be deduced the Rule delivered in this *Problem*.

COROLLARY I.

AND by the same way of arguing, will be easily found the Value of an *Annuity* upon as many *Lives* as may be

be assigned ; for if from the Sum of the Values of all the *Lives* be subtracted the Sum of the Values of all the *joint Lives* combined Two and Two, and to the *Remainder* be added the Sum of the Values of all the *joint Lives* combined Three and Three ; and from this last Sum be subtracted the Sum of the Values of all the *joint Lives* combined Four and Four, and so on, (adding and subtracting alternately) the last Result will exhibit the Value of an *Annuity* upon the last of the *given Lives*.

AND as the Method taken in the *Solution* of the Fourth and Sixth *Problems*, for finding the Values of Two and Three *joint Lives* may, by bare Inspection, be carried on for finding the Values of as many *joint Lives* as may be assigned, so it appears that there remains no Difficulty in the *Solution* of the Case proposed in the present *Corollary*.

COROLLARY II.

IF all the *Lives* be equal, the *Operations* necessary to the *Solution* of the *Problem* will considerably be shortened, for let M' , M'' , M''' , M'''' be the respective *Values* of one *single Life*, two, three, and Four *joint Lives*; then the *Value* of an *Annuity* upon the longest of the four *Lives* will be

$4M' - 6M'' + 4M''' - M''''$; in which the *Numerical Quantities* are the *Coefficients* of a *Binomial*, whose *Index* is equal to the *Number* of the *Lives* given.

R E M A R K.

TO the End that the *Readers* may be freed from any *Scruple* they may entertain, that the converting *real Lives* into *fictitious* ones, and then combining them together, from thence to deduce the *Values* of *Annuities* upon the *real Lives*, may perhaps not be altogether to be trusted to; I shall here annex a *Calculation* of an *Annuity* upon
on

on two *Lives* whose Decrements are in *Arithmetic Progression*; which consequently very nearly agree with the Values as deduced from the *Tables*; and then compare the Result with the Value of an *Annuity* upon the two *Lives* considered as *fictitious*.

LET the Difference between the first Age and 86 be supposed $= n$, and the Difference between the second and 86 $= p$; let the Value of an *Annuity* certain for the Number of Years n be supposed $= N$, and the Value of an *Annuity* certain for the Number of Years $p = P$; then the Value of an *Annuity* upon the two *Lives* will be

$$\frac{1 - \frac{r}{n} \overline{N+P}}{r-1} + \frac{2rp - \overline{Pr}r - Pr}{r-1^2}.$$

SUPPOSING now $n = 50$, $p = 40$, and $r = 1.05$, the Value of an *Annuity* upon the two *Lives* will be found to be 14.53, as may easily be derived from the foregoing *Cannon*.

AGAIN, if n be $= 50$, and $p = 40$, then by the first *Problem* the Value of an *Annuity* upon the first *Life* will

will be 12.332, and the Value of an *Annuity* upon the second will be 10.191; let these two *Lives* be considered as *fictitious*, then by the Fifth *Problem* the Value of an *Annuity* upon both will be found to be 14.55; which differs from the preceding Value only by 0.02.

IF any Person has the Curiosity of calculating the Value of an *Annuity* upon two or three *Lives*, as it may be deduced from the *Tables*, the easiest way of doing it will be first to calculate the Values of the *single* and of the *joint Lives*, and then to combine them in the same manner as has been done for the *fictitious Lives*; we have shewn already how to compute the Values of the *single Lives*, as deduced from the *Tables*; now in order to estimate in the same manner the Values of the *joint Lives*, nothing more is requisite than to follow the *Steps* of the Method employed in estimating the Values of the *joint Lives* considered as *fictitious*; with this Difference, that instead of the Quantity x , there ought to be written a *Fraction*, whose *Numerator* be the Number

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ber in the *Tables*, answering to the next Year above the Age given, and the *Denominator* the Number answering to the Age given; and instead of $x x$ ought to be written a *Fraction*, whose *Numerator* be the Number in the *Tables*, answering to the second Year above the Age given, and the *Denominator* still the Number answering to the Age given, and so on; the like being also to be done for z and y , then instead of the Terms of *Geometric Progression*, there will be a *finite* Number of Terms to be summed up by vulgar Addition.

Of REVERSIONS.

PROBLEM VII.

TO find the Value of the Reversion of one Life after another.

SOLUTION.

FROM the Value of the two Lives, subtract the Value of that Life which is in Possession, there will
H remain

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remain the Value of the *Life* in Reversion ; thus if the Value of the *Life* in Possession be worth Ten Years Purchase, and the Value of the other *Life* be worth 8 Years Purchase, then by *Problem V.* the Value of the two *Lives*, (by which is meant the Value of an *Annuity* upon the longest of the two *Lives*) will be 12.4, from which subtracting the Value of the *Life* in Possession, viz. 10, there will remain 2.4 for the Value of the Reversion, but if the second *Life* be in Possession, then the Reversion will be worth 4.4.

PROBLEM VIII.

TO find the Value of the Reversion of one Life after two.

SOLUTION.

FROM the Value of the three *Lives*, subtract the Value of the two *Lives* in Possession, there will remain the Value of the *Life* in Reversion

version ; thus if the three *Lives* be worth 12, 10 and 8 Years Purchase, and the first *Life* be in Reversion, then from the Value of the three *Lives*, viz. 15.396 subtracting the Value of the two *Lives* in Possession, viz. 12.4 the Remainder, viz. 2.996 will be the Value of the first *Life* in Reversion ; but if the second *Life* be in Reversion, it will be found to be 1.618, and if the third be in Reversion, it will be but 0.808.

PROBLEM IX.

TO find the Value of the Reversion of two *Lives* after one.

SOLUTION.

FROM the Value of the three *Lives*, subtract the Value of the *Life* in Possession, there will remain the Value of the two *Lives* in Reversion ; thus if the three *Lives* be respectively 12, 10 and 8 Years Purchase, the Value of the three *Lives* will be 15.396 from which

which subtracting severally the three Lives 12, 10 and 8, the three Remainders, viz. 3.396, 5.396, 7.396 will express the respective Values of the second and third after the first, of the first and third after the second, and of the first and second after the third.

PROBLEM X.

TO find the Value of the Reversion of one Life, after two joint Lives.

SOLUTION.

FROM the Value of Life in Expectation, subtract the Value of the two joint Lives in Possession, there remains the Value of the Reversion.

PROBLEM XI.

TO find the Value of the Reversion of two joint Lives after one.

SOLU-

SOLUTION.

FROM the Value of the two *joint Lives* in Expectation, subtract the Value of the three *joint Lives*, there remains the Value required.

THUS, suppose three Persons, A, B, C; that C is in Possession, and that A expects the Reversion of an *Annuity*, after the *Life* of C, to be by him enjoyed no longer than whilst he and B continue in being; then the Value of such Reversion to A, will be the Value of the *joint Lives* of himself and B, lessened by the Value of the three *joint Lives*, and that Reversion to B, would be of the same Value as it is to A.

PROBLEM XII.

IF A, B, C agree amongst themselves to buy an *Annuity* to be by them equally divided, whilst they live together, then to be divided equally between the two next
Survivours,

Survivours, then to belong intirely to the last Survivor for Life, to find what each of them ought to contribute towards the Purchase.

SOLUTION.

LET the whole *Annuity* be 1*l.* then,

1° A expects an *Annuity* of $\frac{1}{3}$ *l.* upon the three joint *Lives* of himself, B and C; the Value of which Expectation is to be found by *Problem V.*

2° HE expects the Reversion of $\frac{1}{2}$ *l.* a Year after the *Life* of C, so long as he and B are in being; the Value of which is to be found by *Problem XI.*

3° HE expects the Reversion of $\frac{1}{2}$ *l.* a Year after the *Life* of B, to be by him enjoyed so long as he and C are in being; the Value of which is found also by *Problem XI.*

4° HE expects the Reversion of 1*l.* a Year, to be by him enjoyed for his *Life*, after the *Lives* of B and C; the Value of which Expectation, may be determined by *Problem VIII.*

W H E R E-

WHEREFORE adding all these Expectations together, the Sum will be the Share which A, ought to contribute towards the Purchase.

AND in the like Manner, may the Shares of B and C be determined.

BUT all the preceding Operations may be contracted into a narrower Compass, as follows :

FROM *Unity* subtract $\frac{1}{2}$ the Sum of the Values of the *single Lives* of B and C ; and to Remainder add $\frac{1}{3}$ of the Value of their *joint Lives*, the Sum multiplied by the Value of the *Life* of A, the *Product* will be the Share which A ought to contribute,

And the like may be done for B and C.

PROBLEM XIII.

IF B and C, expect the Reversion of an Annuity after the Life of A, and this Annuity is to be by them equally enjoyed so long as they both continue in being, then entirely to belong to the last Survivor for Life ;

Life; to find what Proportion of the Purchase both B and C ought to contribute.

SOLUTION.

LET the whole Annuity be 1.

1^o B expects after the Life of A, the Reversion of an Annuity to be by him enjoyed as long as he and C both continue in being; the Value of which Expectation is to be found by Problem XI.

2^o HE expects after the Lives of A and C, the Reversion of an Annuity of 1 l. for his Life, the Value of which Expectation is found by Problem VIII.

WHEREFORE adding these two Expectations together, the Sum will be the Share which B ought to contribute, towards the Purchase of the Reversion.

AND in the like Manner may the Share of B be determined.

THE

THE Contractions that may be made in these Operations are so inconsiderable that they are not worth mentioning.

O F
S U C C E S S I V E L I V E S .
P R O B L E M X I V .

IF A enjoys an Annuity for his Life, and at his Decease has the Nomination of a Successor who is likewise to enjoy the Annuity for his Life ; to find the Value of the two Successive Lives.

S O L U T I O N .

LET the Values of the Lives considered separately be respectively M and P ; r the Rate of Interest, and a the Interest of One Pound ; then the Value of the two Successive Lives will be $\frac{r - 1 - aM \times 1 - aP}{ra}$; which may be thus expressed.

I

MUL-

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MULTIPLY each given *Life* by the Interest of One Pound, and subtract each *Product* from *Unity* ; let the two *Remainders* be multiplied together, and the *Product* subtracted from the *Rate of Interest* ; then the *Remainder* being divided by the *Rate of Interest* multiplied by the Interest of One Pound, the *Quotient* will give the Value of the two *Successive Lives*.

PROBLEM XV.

THREE Lives M, P, Q, being given in Succession, to find their Value.

SOLUTION.

LET r and a stand for the same Quantities as in the preceding Problem ; then the Value sought will be

$$\frac{rr - \overline{1-a}M \times \overline{1-a}P \times \overline{1-a}Q}{rra} ;$$

which may be thus expressed.

MULTIPLY each given *Life* by the Interest of One Pound, and subtract the several

several *Products* from Unity ; let the three *Remainders* be multiplied together, and the *Product* subtracted from the Square of the Rate of Interest, then the *Remainder* being divided by the Square of the Rate of Interest multiplied by the Interest of One Pound, the *Quotient* will be the Value sought.

AND the same Rule may be extended to four, five or as many *Lives* as may be assigned ; the *Divisor* for four *Lives* being to be the *Product* of the Cube of the Rate of Interest, by the Interest of One Pound, and so on.

PROBLEM XVI.

IF there be three equal *Lives*, and A or his *Heirs* are to have the Sum £ paid them upon the *Vacancy* of any of those *Lives*, what is the *Expectation* of A worth in present Money.

SOLUTION.

LET the Value of one *Life* increased by Unity be raised to its Cube.

I 2

2° MUL-

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2° MULTIPLY the Value of one Life by the Rate of Interest, and let the Product be raised to its Cube.

3° FROM the first Cube subtract the second, and keep the Remainder.

4° MULTIPLY the first Cube by the Rate of Interest, and from the Product subtract the second Cube, then keep the Remainder.

5° MULTIPLY the first Remainder by the Sum to be paid, and divide the Product by the second Remainder; then the Quotient will express the present Value of the Expectation required.

THUS suppose that each Life is worth Ten Years Purchase, and that 100 l. is to be paid upon the Vacancy of any one of the Lives; to find the Value of that Sum in present Money.

11. Value of one Life increased by Unity.

Its Cube 1331.

10. Value of one Life.
multiply by 1.05 Rate of Interest.

Pro-

Product 10.5
 its Cube 1157.625 which subtracted
 from the preceding Cube

First Remain^{er}. 173.375

First Cube 1331.

Rate of Int. 1.05

Product 1397.55

subtract 1157.625 second Cube.

2d Remaind. 239.925

1st Remaind. 173.375

Multiply by 100. Sum to be paid.

Product 17337.5

239.925) 17337.500 (74. present Value of the Expectation.

DEMONSTRATION.

LET x be the Probability that any one of the *Lives* may continue one Year.

THEN, the Probability that all the *Lives* may continue one Year is x^3 , therefore the Probability that some of them may fail the first Year is $1 - x^3$.

THE

THE Probability that all the *Lives* may continue one Year, and then that some of them may fail the next Year is $x^3 \times \overline{1 - x^3}$.

THE Probability that all the *Lives* may continue two Years, and then that some of them may fail on the third Year is $x^6 \times \overline{1 - x^3}$.

WHEREFORE the Value of A's Expectation is $\frac{1}{r} + \frac{x^3}{rr} + \frac{x^6}{r^3} + \frac{x^9}{r^4}$, &c. $\times \overline{1 - x^3} \times \int$ the Sum of which is $\frac{\int}{r - x^3} \times \overline{1 - x^3}$; but $x = \frac{Mr}{M+1}$, therefore the Value of A's Expectation is $\frac{M+1^3 - M^3 r^3}{M+1^3 \times r - M^3 r^3} \times \int$.

PROBLEM XVII.

IF A buys an Annuity upon three equal *Lives*, and he obliges himself that in Case one Life becomes vacant he shall renew it for the Sum *a*; in Case two *Lives* become vacant in the same Year, he shall renew them both for the Sum *b*; and in Case they all become vacant in the same Year, he shall renew them all three for the Sum *c*; to find how

how much it is that the whole Purchase is estimated.

Solution of the first Case.

1° FROM the Value of one *Life* increased by Unity, subtract the Product of the Value of one *Life* by the Rate of Interest.

2° MULTIPLY the Remainder by three times the Square of that Product, and let the new Product be made the Numerator of a Fraction.

3° LET the Value of one *Life* increased by Unity, be raised to its Cube, and multiply that Cube by the Interest of one Pound, then let the Product be made the Denominator of that Fraction.

4° THIS Fraction being multiplied by the Sum agreed to be given for the renewing of one *Life*, the Product will be the present Value of all the Renewals of one *Life*, that may be made for ever.

Solu-

Solution of the second Case.

1° **F**ROM the Value of one *Life* increased by Unity, subtract the Product of the Value of one *Life* by the Rate of Interest.

2° **L**ET the Remainder be raised to its Square, and let that Square be multiplied by three Times the Product above-mentioned, and let the new Product be made the Numerator of a Fraction.

3° **L**ET the Value of one *Life* increased by Unity be raised to its Cube, and multiply that Cube by the Interest of one Pound, then let the Product be made Denominator of the *Fraction* above-mentioned.

4° **T**HIS *Fraction* being multiplied by the Sum agreed to be given for the renewing of two *Lives*, the Product will be the present Value of all the Renewals of two *Lives* that may be made in the same Year for ever.

Solution of the third Case.

1° FROM the Value of One *Life* increased by Unity, subtract the Product of the Value of one *Life* by the *Rate of Interest*.

2° LET the Remainder be raised to its Cube, and let that Cube be made Numerator of a Fraction.

3° LET the Denominator of that Fraction be the same as the Denominators of the Two preceding Cases.

4° THE Fraction being multiply'd by the Sum agreed to be given for the renewing of Three *Lives*, the Product will be the present Value of all the Renewals of three *Lives* that may be made in the same Year for ever.

AND the Three several Sums resulting from the *Solution* of these Three Cases, being added with the present Value of the Three *Lives*, the Sum of the whole will be the Value of the whole Purchase.

DEMONSTRATION.

LET x represent the Probability of each *Life's* continuing one Year, M the Value of one *Life* and r the Rate of Interest.

THE Probability that Two of the Three *Lives* may continue one Year, and the third become vacant, is

$$3 \times x \times \overline{1 - x}.$$

WHEREFORE the Sum, which A may happen to give at the Year's end for renewing one *Life* estimated in pre-

sent Money is $\frac{3 \times x \times \overline{1 - x}}{r} a.$

SUPPOSE that *Life* renewed, then the Probability that one of the Three *Lives* may become vacant the next Year is again $3 \times x \times \overline{1 - x}.$

WHEREFORE the Sum which A may happen to give at the End of the second Year, for renewing one *Life* estimated in present Money, is

$$\frac{3 \times x \times \overline{1 - x}}{rr} a.$$

IN the like Manner, the Sum which *A* may happen to give at the End of the third Year estimated in present Money, is $\frac{3xx \times \overline{1-x}}{r^3}$.

FROM whence it follows, that all the Sums which *A* may happen to give from Time to Time for the renewing of one Life estimated in present Money, are

$$\frac{3xx \times \overline{1-x}}{r-1} a = \frac{3MMrr \times \overline{M+1-Mr}}{(M+1)^3 \times r-1} a,$$

THE probability that one of the Three Lives may continue one Year, and the other two become vacant, is $3xx \times \overline{1-x}^2$.

WHEREFORE the Sum which *A* may happen to give at the Year's End for the renewing of two Lives, estimated in present Money, is worth $\frac{3xx \times \overline{1-x^2}}{r} b$

AND for the same Reason, the Sum which he may happen to give at the end of two Years for the renewing of Two Lives, estimated in present Money, is $\frac{3xx \times \overline{1-x^2}}{rr} b$, and so on.

FROM whence it follows, that all the Sums which *A* may happen to give from Time to Time, for the renewing of Two Lives, estimated in present Money, are

$$\frac{3x \times 1 - x}{r - 1} b = 3Mr \times \frac{M + 1 - Mr^3}{M + 1^3 \times r - 1} b.$$

THE Probability that the Three Lives may be vacant the first Year, is $\frac{1}{1 - x^3}$.

FROM whence it follows that all the Sums which *A* may happen to give from Time to Time for the renewing of Three Lives, estimated in present Money, are $\frac{M + 1 - Mr^3}{M + 1^3 \times r - 1} c$.

R E M A R K.

IF one Life be estimated at Nine Years Purchase; then the Value of two equal Lives is 12.3111, and the Value of three equal Lives is 14.0281: Wherefore what *A* ought in Justice to give for the renewing of One Life is 1.7170, what he ought to give for the renewing of Two is 5.0281; and what he ought to give for the renewing of Three is 14.0281 if he loses the last

last Year's Rent ; or 15.0281 if he receives it. Let us therefore suppose that in the foregoing Problem, $a = 1.7170$, $b = 5.0281$, $c = 15.0281$; then what A ought in Justice to give in present Money for the renewing of One Life for ever, is 5.0600 ; what he ought to give for the renewing of Two for ever, is 0.8624 ; and what he ought to give for the renewing of Three for ever is, 0.0500, the Sum of all which, viz. 5.9724 being added to 14.0281 Value of the three Lives, the Sum is 20, which makes up the whole Purchase of an Annuity bought for ever, Interest being supposed at 5 per Cent.

SUPPOSING now that one Life being estimated at Nine Years Purchase as before, what is given for the renewing One, Two and Three Lives should be respectively 3, 5 and 15.0281 Years Purchase, or that instead of renewing the Three Lives, whenever they happen to be vacant in the same Year, the right of the Annuity should return to the first Proprietor ; then it will be found by the Rules above delivered that the whole

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whole Purchase is estimated at 24.1197
Years Purchase.

PROBLEM XVIII.

THE same Things being given as in the preceding Problem, suppose it be agreed that instead of renewing One Life for the Sum *a*, whenever it happens to be vacant, the Purchaser shall constantly pay that same Sum *a* at a Certainty, at equal Intervals of Time; to find at what Intervals it ought to be paid.

SOLUTION.

1° **F**ROM the Value of One Life increased by Unity, subtract the Product of the Value of one Life multiplied by the Rate of Interest.

2° **MULTIPLY** the Remainder by three Times the Square of that Product, and let the new Product be made the Numerator of a Fraction.

3° **LET** the Value of One Life increased by Unity be raised to its Cube, and let that Cube be made the Denominator of the Fraction.

4° **TO**

4° TO the Quotient of the Numerator divided by the Denominator, add the Interest of One Pound, then find the Logarithm of that Sum.

5° FROM that Logarithm subtract the Logarithm of the Fraction before-mentioned, and let the Remainder be divided by the Logarithm of the Rate of Interest; the Quotient will be the Number of Years sought.

DEMONSTRATION.

THE first Renewal of One *Life* may happen to be at the End of the first Year, whereof the Probability is $3 \times x \times \overline{1-x}$; let that Quantity for Brevity's Sake be called S ; therefore the present Value of the Renewal of one *Life* arising from the Expectation of its happening in the first Year, is $\frac{S}{r} a$.

THE Probability of the first Renewal's being in the second Year, is $S \times \overline{1-S}$; wherefore the present Value of the first Renewal arising from the

the Expectation of its happening in the second Year is $\frac{S \times \overline{1 - S}}{rr} a$.

AND for the same Reason, the present Value of the first Renewal arising from the Expectation of its happening in the third Year is $\frac{S \times \overline{1 - S^2}}{r^3} a$, and so on.

FROM whence it follows, that the present Value of the first Renewal arising from the Expectation of its happening at any Time is expressible by the following *Geometric Progression*, viz.

$$\begin{aligned} & \frac{S}{r} a + \frac{S \times \overline{1 - S}}{rr} a + \frac{S \times \overline{1 - S^2}}{r^3} a \\ & + \frac{S \times \overline{1 - S^3}}{r^4} a, \text{ \&c. whose Sum} \\ & \text{is } \frac{S}{r - 1 + S} a. \end{aligned}$$

LET now, n be the Number of Years supposed to be intercepted between the the present Time, and the Time of the first Renewal; then the Quantity

Quantity $\frac{S}{r-1+S}$ a before found being put out at Interest for that Number of Years n , ought to produce the Sum a ; then according to the Rules of Compound Interest, $\frac{S}{r-1+S} a r^n = a$ or $\frac{S}{r-1+S} r^n = 1$. Wherefore $n = \frac{\text{Log. } r-1+S - \text{Log. } S}{\text{Log. } r}$; then instead of S , writing its Value $3 \times x \times 1 - x$, and instead of x writing its Value $\frac{Mr}{M+1}$, the Reason of the Rule given in Words at length, will evidently appear.

COROLLARY I.

THE present Value of the first Renewal of one *Life* being $\frac{a}{r^n}$, the present Value of the Second will be $\frac{a}{r^{2n}}$; of the Third $\frac{a}{r^{3n}}$, &c. Wherefore the present Value of all the Renewals for ever is $\frac{a}{r^n - 1}$, and in the room of r^n , substituting its Value

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lue $\frac{r-1+S}{S} a$, the present Value, of all the Renewals of one *Life*, will be $\frac{S}{r-1} a = \frac{3 \times \times \times 1 - \infty}{r-1} a$, which is the same Conclusion that was drawn in the *Solution* of the preceding *Problem* by a Method different from this.

COROLLARY II.

IN the same manner may the renewing of Two or Three *Lives* be compounded for by equal Payments made at equal Intervals of Time.

COROLLARY III.

AND these Three Setts of Intervals may be reduced to One single Interval, of which the Duration may be determined.

Of the Expectation of Life and the Probability of Survivourship.

THE Questions that may be formed about the Expectation of *Life*, and the Probability of Survivourship

ship being independent from the Consideration of the *Rate of Interest*, we have thought fit to treat this Subject by it self, and shew how to make a very near Estimation of both by Dr. HALLEY's *Tables*, or any other *Tables* that may happen to be constructed from Observations.

BUT because, as I have already observed somewhere else, the Decrements of *Life* are for any small Interval of Time nearly in *Arithmetic Progression*, it will not be improper to consider what would be the Consequence of an Hypothesis that should make the Decrements of *Life*, throughout the whole extent of it, to be in *Arithmetic Progression*, and then shew how such an Hypothesis ought to be corrected from the *Tables of Observations*.

DEFINITION.

I Call that the Complement of *Life*, which is the Time comprehended between an Age given, and the Extremity of Old Age.

L 2

THUS

THUS if the Age be 50, and the Extremity of Old Age be 86, the Complement of *Life* is 36.

HYPOTHESIS.

A B C D E F G

S

LET it be supposed that the Complement of *Life* A S being divided into an infinite Number of equal Parts, representing Moments; the Probabilities of living from A to B, from A to C, from A to D, &c. are respectively proportional to the several Complements S B, S C, S D; in so much that these Probabilities may respectively be represented by the *Fractions*, $\frac{SB}{SA}$, $\frac{SC}{SA}$, $\frac{SD}{SA}$, &c. This Hypothesis being admitted, the following *Corollaries* may be deduced from it.

COROLLARY I.

THE Probability of *Life's* failing in any Interval of Time A F, is measured by the *Fraction* $\frac{FA}{SA}$.

CO-

COROLLARY II.

WHEN the Interval AF is once past, the Probability of *Life's* continuing from F to G is $\frac{SG}{SF}$ (for at F , the Complement of *Life* is SF) and the Probability of its failing in the Interval FG is $\frac{FG}{SF}$.

COROLLARY III.

THE Probability of *Life's* continuing from A to F , and then failing from F to G is $\frac{SF}{SA} \times \frac{FG}{SF}$
 $= \frac{FG}{AS}$

COROLLARY IV.

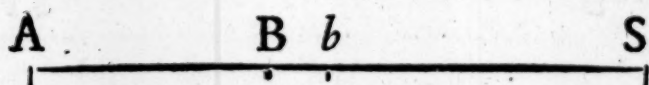
THE Probabilities of *Life's* failing in any Two or more equal Intervals of Time assigned between A and S are exactly the same, the Estimation being made at A , considered as the present Time.

PRO-

PROBLEM XIX.

TO find how long it may be expected that a Life of a given Age may continue in Being.

SOLUTION.



IN the Interval AS take any Term B at Pleasure, and next to it another Term b , so that the Distance Bb may represent a Moment of Time; let the whole Interval AS be called n , the Interval AB , z , and the Moment Bb , o .

WHEN the *Life* given has continued from A to B , the Thing immediately expected is the Moment Bb , but the Probability of its continuing from A to B is $\frac{n-z}{n}$ by Hypothesis, and the Moment expected is o .

THERE-

THEREFORE the Value of the Expectation of the Moment o is $\frac{n-z}{n}$

$\times o$: Consider now $\frac{n-z}{n}$ as the Ordinate of a Curve raised perpetually upon the Abscissa z ; then the Area of that Curve, viz. $\frac{nz - \frac{1}{2}z^2}{n}$ will

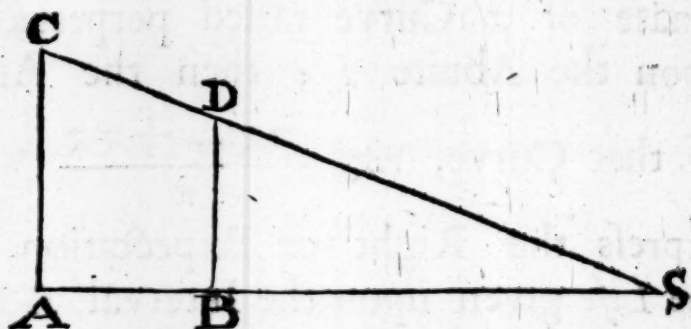
express the Right or Expectation of the *Life* given upon the Interval AB : Let now n be written instead of z , and the Value of the *Fraction* will become $\frac{1}{2}n$.

WHEREFORE the Number of Years that a *Life* given may expect to continue in Being is equal to the Number of Years contained in half the Complement of the said *Life*; that if the Age given be 40, and the Extremity of Old Age be supposed at 86, the Complement of the *Life* is 46 whereof the Half 23 is the Number of Years it may be expected to continue after 40.

BUT if that *Life* once reaches 63, then it may expect $11 \frac{1}{2}$ Years more; and

and if it reach $74 \frac{1}{2}$, then may expect $5 \frac{3}{4}$ more, and so on.

COROLLARY I.



SUPPOSE the Parallels AC and BD terminated in C and D to represent the Number of Persons of two different Ages according to the *Table of Observations*, and AB the Number of Years between those Ages; draw the Line CD till it meet the Line AS in S, let AS be supposed = n , AB = z , AC = a , BD = b : now since the Triangles CAS, DBS are similar, it follows that

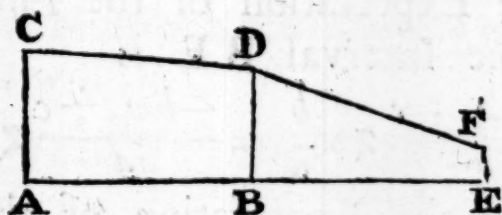
$$AC - BD, AB :: AC, AS, \text{ that is, } a - b, z :: a, n$$

THERE

THEREFORE $n = \frac{az}{a-b}$. But the Expectation of the *Life* at A upon the Interval AB has been found to be $\frac{nz - \frac{1}{2}zz}{n}$; Wherefore writing

$\frac{az}{a-b}$ in the room of n , that Expectation will be found to be $\frac{\frac{1}{2}a + \frac{1}{2}b}{a} z$.

COROLLARY II.



LET also AC, BD, EF; the the Number of Persons of three different Ages as taken from the *Tables of Observations*, which Numbers we suppose respectively denominated by a, b, c ; let AB, BE represent Two equal Intervals of Time, intercepted between those Ages, which Interval we suppose denominated by z .

M

THIS

THIS being supposed, the Expectation of the *Life* at A upon the Interval

AB is $\frac{\frac{1}{2}a + \frac{1}{2}b}{a} z$, by the preceding

Corollary; the Expectation of the same *Life* upon the next Interval BE, will

be $\frac{\frac{1}{2}b + \frac{1}{2}c}{b} z$, when once the Ex-

tremity of the Interval AB is attained; but the Probability of attaining it, is

measured by the *Fraction* $\frac{b}{a}$; where-

fore the Expectation of the *Life* at A upon the Interval BE is

$= \frac{\frac{1}{2}b + \frac{1}{2}c}{b} z \times \frac{b}{a} = \frac{\frac{1}{2}b + \frac{1}{2}c}{a} z$, and

therefore the Expectation of the *Life* at A upon the whole Interval AE

$= \frac{\frac{1}{2}a + b + \frac{1}{2}c}{a} z$.

COROLLARY III.

AND therefore if the Complement of *Life* be divided into equal Intervals, and we take out of the *Table of Observations* the Numbers terminating those Intervals, and from the

Sum

Sum of all those Numbers, we subtract one Half of the First, and then multiply the Remainder by One of the equal Parts, and lastly divide the Product by the Number answering to the Beginning of the First Interval; the Quotient will express the Number of Years which compose the Expectation of the Life given.

THUS if we consider a Life of 50, and divide its Complement, or Difference between 50 and 86, into several equal Intervals of Six Years each; then the Numbers terminating those Intervals will be

Years.	Persons.
50	346
56	282
62	222
68	162
74	98
80	41
86	0
	1151 Sum
From which subtract	173 one Half
of the first.	
M 2	Re-

	<i>Remainder</i>	978	
	Multiply by	6	One of the
equal	Parts.		
	<i>Product</i>	5868	

which *Product* being divided by 346 Number answering to the Beginning of the first Interval; then the *Quotient* 16.95 or 17 nearly will express the Number of Years which a *Life* of 50 may reasonably expect.

COROLLARY IV.

BUT if the Age be such, that having divided the Complement of *Life* belonging to it, into any Number of Parts taken at Pleasure, as many of them as possible containing the same Number of Years, such as 6, it should happen that the last of the equal *Divisions* could not be terminated by 86, as will happen for a *Life* of 40.

THEN having added together all the Numbers terminating the equal Intervals; from the Sum of the whole, subtract the half Sum of the greatest and

and least of them, then multiply the *Product* by one of the equal Parts, and to the *Product* add another *Product* made of one Half the Number terminating the last of the equal Intervals, by the last odd Interval; then the Sum being divided by the Number answering to the Beginning of the first Interval, the *Quotient* will express the Number of Years which the *Life* given may expect.

THUS supposing a *Life* of 40, the Operation will be as follows,

Years.	Persons.	
40	455	
46	387	
52	324	
58	262	
64	202	
70	142	
76	78	
82	28	
86	0	
	1868	Sum
	subtract 236	half Sum
of 445 and 28.		

Re-

Remainder 1632
 which multiplied by 6 one of equal Parts.

<i>Product</i>	9792	
to which add	56	<i>Product</i> of
the Number 28 by the last Interval 4.		
<i>Sum</i>	9848	

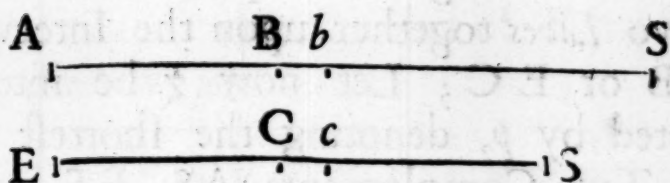
Divide this Sum by 445 Number answering to 40 beginning of the first Interval; the *Quotient*, viz. 22.13 will express the Number of Years due to a *Life* of 40.

P R O B L E M XX.

TWO Lives being given, *whether equal or unequal*; To find how many Years they may be expected to continue together.

SOLU-

SOLUTION.



LET the Complements, of the Lives, A S, E S be respectively supposed equal to n and p ; take upon A S and E S Two equal Intervals A B, E C.

THE Probability of the first Life's continuing from A to B is $\frac{n-z}{n}$; the Probability of the second Life's continuing from E to C, is $\frac{p-z}{p}$; wherefore the Probability of their both continuing during the Interval A B or E C is measured by the Product of those Two Fractions, viz.

$$\frac{np - nz - pz + zz}{np}.$$

Let this Product be considered as the Ordinate of a Curve whose Abcine is z , then the Area of that Curve, viz.

$$npz$$

$$\frac{n p z - \frac{1}{2} n z z - \frac{1}{2} p z z + \frac{1}{3} z^3}{n p}$$

expresses the Right or Expectation of the Two *Lives* together upon the Interval A B or E C ; Let now z be interpreted by p , denoting the shortest of the Two Complements A S, E S, and then the Expectation of both *Lives* together upon the shortest Interval A S will be measured by the *Frac-*

$$\text{tion } \frac{n p p - \frac{1}{2} n p p - \frac{1}{2} p^3 + \frac{1}{3} p^3}{n p}$$

$$= \frac{1}{2} p - \frac{\frac{1}{6} p p}{n} ; \text{ which may be expressed as follows.}$$

TAKE the Square of the shortest Complement, and divide it by the greatest, the sixth Part of the *Quotient* being subtracted from one Half of the shortest Complement, the *Remainder* will express the Number of Years sought.

THUS supposing a *Life* of 40, and another of 50, the shortest Complement will be 36 ; the greatest, 46 ; the Square of the shortest Complement will be 1296 ; which being divided by the greatest 46, the *Quotient* will be
28.175,

28.175, whereof the 6th Part, viz. 4.69 being subtracted from half the shortest, viz. 18, the Remainder 13.31 will express the Number of Years due to the two joint Lives.

COROLLARY.

IF the two *Lives* be equal, the Number of Years due to their joint Continuance, is expressible by the Third Part of their common Complement.

THUS supposing each *Life* to be 50, the Number of Years due to the two joint *Lives* is 12.

PROBLEM XXI.

ANY Number of *Lives* being given whether equal or unequal, to find how many Years they may be expected to continue together.

SOLUTION.

1° **T**AKE $\frac{1}{2}$ the shortest Complement.

N

2° TAKE

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2° TAKE $\frac{1}{6}$ Part of the Square of the shortest, which divide successively by all the other Complements; then add all the *Quotients* together.

3° TAKE $\frac{1}{12}$ Part of the Cube of the shortest Complement, which divide successively by the Products of all the other Complements taken Two and Two.

4° TAKE $\frac{1}{20}$ Part of the Biquadrate of the shortest Complement, which divide successively by the *Products* of all the other Complements taken Three and Three, and so on.

5° THEN from the Result of the first *Operation*, subtract the Result of the second, to the Remainder add the Result of the Third; from the Sum subtract the Result of the fourth, and so on.

6° THE last Quantity remaining after these alternate Subtractions and Additions will be the Thing required.

N. B. THE Divisors 2, 6, 12, 20, &c. are the Products of 1 by 2; of 2 by 3; of 3 by 4; of 4 by 5, &c.

COROL-

COROLLARY.

IF all the *Lives* be equal, add Unity to the Number of *Lives*, and divide their common Complement by that Number thus increased by Unity, and the Quotient will always express the Time due to their joint Continuance.

PROBLEM XXII.

TWO *Lives* being given, to find the Number of Years due to the longest.

SOLUTION.

FROM the Sum of the Years due to each *Life*, subtract the Number of Years due to their joint Continuance, the *Remainder* will be the Number of Years due to the longest.

THUS supposing a *Life* of 40, and another of 50; the Number of Years due to the *Life* of 40, is 23; the Number of Years due to the *Life* of 50, is 18; from the Sum of 23 and 18, viz.

N 2

41,

41, subtract 13.31 due to their joint Continuance; the Remainder 27.69 will be the Time due to the longest.

COROLLARY.

BUT if the *Lives* be equal, then $\frac{2}{3}$ of their common Complement will be the Number of Years due to the longest.

THUS supposing Two *Lives* of 50, it may be expected that the longest of the Two will continue 24 Years.

PROBLEM XXIII.

THREE *Lives* being given, to find the Number of Years due to the longest.

SOLUTION.

FROM the Sum of the Years due to the single *Lives* subtract the Sum of the Years due to all the joint *Lives* combined Two and Two; then to the Remainder add the Number of Years due to the three joint *Lives*; and this

this last Sum will be the Number of Years due to the longest of the Three Lives.

COROLLARY.

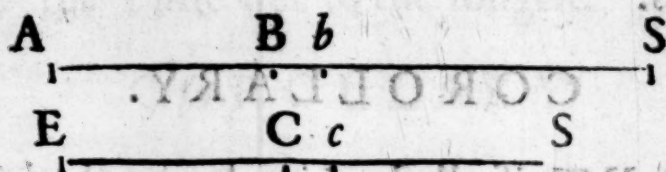
BUT if all the *Lives* be equal, then $\frac{1}{4}$ of their common Complement will be the Number of Years due to the longest.

AND Universally, supposing any Number of equal *Lives*; take such Part of their common Complement as may be exprest by a *Fraction* whose Numerator be equal to the Number of *Lives*, and Denominator, the same Number increased by Unity, and that Part thus taken will answer the Number of Years due to the longest.

PROBLEM XXIV.

TWO *Lives* being given, to find the Probability of one of them, fixt upon, surviving the other.

SOLUTION.



LET the Complements of the Two Lives be respectively AS and ES , upon which take Two equal Intervals AB , $EC = z$; as also Two Moments Bb , $Cc = o$.

THE Probability of the first Life's continuing from A to B or beyond it, is measured by the Fraction $\frac{n-z}{n}$;

the Probability of the Second's continuing from E to C and then failing in the Interval Cc , is measured by the Product of the Two Fractions $\frac{p-z}{p}$

and $\frac{1}{p-z}$, that is by $\frac{o}{p}$; therefore the Probability of the first Life's continuing during the Time AB or beyond it, and of the Second's failing just at the End of that Time, is $\frac{n-z}{n} \times \frac{o}{p}$: Let the Quantity $\frac{n-z}{np}$ be

be considered as the Ordinate of a Curve whose Abcisse is z ; then the Area of it, viz. $\frac{n z - \frac{1}{2} z z}{n p}$ will ex-

press the Probability of the first *Life's* continuing during any Interval of Time z or beyond it, and of the second's failing any Time before or precisely at the attaining the End of that Interval. Let now p be written instead of z , and then the Probability of the first *Life's* surviving, the second will be

$$\frac{n p - \frac{1}{2} p p}{n p} = 1 - \frac{\frac{1}{2} p}{n}.$$

FROM the foregoing Conclusion we may immediately infer that the Probability of the second *Life's* surviving the first will be $\frac{\frac{1}{2} p}{n}$, which may also be proved thus.

THE Probability of the second *Life's* continuing during any Interval of Time z or beyond it, and of the first's failing any Time before or at the attaining that Interval is $\frac{p z - \frac{1}{2} z z}{p n}$; let now p be written instead of z , then the Pro-

Probability of the second *Life's* surviving the first will be $\frac{pp - \frac{1}{2} p p}{n p} = \frac{\frac{1}{2} p}{n}$ as before.

FROM whence the following Rule may be derived.

FROM the greater Complement subtract Half of the least; then the Proposition of the *Remainder* to Half the least Complement, will express the Proposition of the Chances which the youngest *Life* has to survive the oldest, and which the oldest has to survive the youngest.

THUS supposing Two *Lives*, viz. of 50 and 62 the Chances of Survivorship are respectively as 24 to 12, or as 2 to 1.

TO adapt this Rule to the *Table* of *Observations*, we may proceed as follows.

1° DIVIDE the Complement of the oldest *Life* into any Number of Intervals, whether equal or unequal.

2° DIVIDE also the Complement of the youngest *Life* into the like Number of Intervals each respectively equal

to

to the Intervals belonging to the oldest *Life*, neglecting the Overplus.

3° TAKE out of the *Tables* the Numbers terminating the Intervals given, and that for both the given *Lives*.

4° MULTIPLY the Sum of the Two Numbers terminating the first Interval belonging to the oldest *Life* by the Difference of the Two Numbers terminating the first Interval belonging to the youngest *Life*.

5° MULTIPLY the Sum of the Two Numbers terminating the second Interval belonging to the oldest *Life*, by the Difference of the Two Numbers terminating the second Interval belonging to the youngest *Life*, and so on.

6° THE Sum of the foregoing *Product* being divided by twice the *Product* of the Two Numbers terminating the Beginning of the Two Sets of Intervals, the *Quotient* will express the Probability of the oldest *Life* surviving the youngest.

AND therefore this *Quotient* being subtracted from Unity, the *Remainder* will express the Probability of the youngest *Life* surviving the oldest.

O

THUS

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THUS suppose a *Life* of 62, and one of 50, let the Difference between 62 and 86 be divided into several parts at Pleasure; let, for Instance, the four first contain each an Interval of 5 Years, and the last an Interval of 4.

TAKE out of the *Tables* the Numbers answering to

62, 67, 72, 77, 82, 86.
viz. 222, 172, 120, 68, 28, 0.

TAKE also out of the *Tables* the Numbers answering to

50, 55, 60, 65, 70, 74.
viz. 346, 292, 242, 192, 142, 98.

THE Sum of the Numbers terminating the first Interval belonging to the oldest *Life* is 394.

THE Difference of the Numbers terminating the first Interval belonging to the youngest *Life* is 54.

WHEREFORE Multiplying 394 by 54, the *Product* will be 21276.

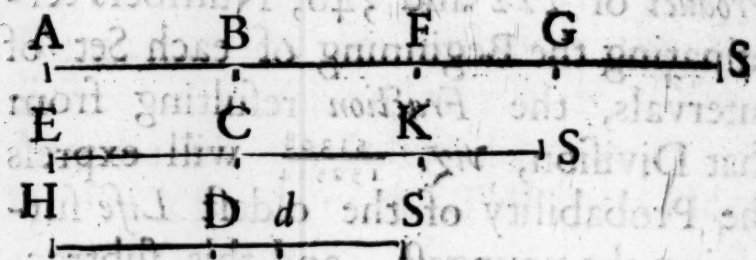
THEN taking in the like Manner the other successive *Products*, we shall have 14600, 9400, 4800, 1232, which *Products* being added with the first before found, viz. 21276, the Sum of all

all will be 51308, and this being divided by 152514 which is twice the *Product* of 222 and 346, Numbers terminating the Beginning of each Set of Intervals, the *Fraction* resulting from that Division, viz. $\frac{51308}{152514}$ will express the Probability of the oldest *Life* surviving the youngest; and this subtracted from Unity, the *Remainder*, viz. $\frac{101206}{152514}$ will express the Probability of the youngest *Life* surviving the oldest; and consequently the Odds of the youngest *Life* surviving the oldest, are as 101206 to 51308, that is very near in the Proportion of 2 to 1, as it had been found from the *Hypothesis* of an equable Decrement of *Life*.

PROBLEM XXV.

THREE Lives being given, to find the Probability of One of them fixed upon, surviving the other Two.

SOLUTION.



LET the Complements of the Ages, viz. AS, ES, HS be respectively called n , p , q .

1^o LET us find the Probability of the first *Life's* surviving the other Two; in order to which, let AG be taken equal to ES, and both AF and EK equal to HS; let there also be taken three equal Distances AB, EC, HD at the End of which the first *Life* may be look'd upon as having survived the other Two, that is, the second *Life* any Time before it reached the Point C, and the third *Life* just at its reaching the corresponding Point D, or immediately after; then there are three Circumstances to be expressed, viz. that of the first *Life's* reaching the Point B, that of the second's failing any Time before

before it comes to the Points C, and that of the third's failing just at its reaching the Point D, or immediately after; all which Circumstances may be represented by the Product $\frac{n-z}{n}$

$\times \frac{z}{p} \times \frac{o}{q}$; let therefore $\frac{n z - z z}{n p q}$

be considered as the Ordinate of a Curve whose Abscisse is z , then the Area of it, viz. $\frac{\frac{1}{2} n z z - \frac{1}{3} z^3}{n p q}$ will

express the Probability of all those Circumstances having happened in some Moment of Time or other comprehended within any indeterminate Space supposed equal to z .

AGAIN, the first *Life* at its reaching the Point B, may be look'd upon as having survived the third *Life* any Time before it reached the Point D, and the second *Life* immediately after its reaching the corresponding Point C, all which Circumstances may be represented by the Product $\frac{n-z}{n} \times \frac{z}{q} \times \frac{o}{p}$,

and therefore $\frac{\frac{1}{2} n z z - \frac{1}{3} z^3}{n p q}$ is once

more

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more the measure of the Probability of all those Circumstances having happened in some Moment of Time or other comprehended within the Space z .

AND therefore $\frac{n z z - \frac{2}{3} z^3}{n p q}$ is the Measure of the Probability of the first *Life's* having survived the other Two within a Space of Time equal to z .

LET now q be written instead of z , then $\frac{n p - \frac{2}{3} q q}{n p}$ will represent the Probability of the first *Life's* having survived the other Two within an Interval of Time equal to the shortest of the Three AS , ES , HS .

LET us now inquire what the Probability is of the first and second *Lives* continuing together during a Time equal to the shortest Interval HS , and then of the first's surviving the second.

THE Probability of the first *Life's* continuing during the Interval AG , and of the second's failing before that Time be expired, is by the preceding Problem

$$1 - \frac{\frac{1}{2} p}{n} = \frac{n p - \frac{1}{2} p p}{n p},$$

THE

THE Probability of the first *Life's* continuing during the Space AF, and of the second's failing before the Expiration of that Time, is $\frac{nq - \frac{1}{2}qq}{np}$.

WHEREFORE, subtracting the second Probability from the First, there will remain $\frac{np - \frac{1}{2}pp - nq + \frac{1}{2}qq}{np}$

expressing the Probability of the First *Life's* having survived the other Two, but not before the End of the shortest Complement be attained.

LET us now add together the Probabilities of the First *Life's* having survived the other Two, both before and after the End of the shortest Complement is attained, and the Sum of those Two Probabilities, viz. $1 - \frac{\frac{1}{2}p}{n} - \frac{\frac{1}{6}qq}{np}$ will be the Probability of the First *Life's* surviving the other Two.

2° IN the same Manner it will be found that the Probability of the second *Life's* surviving the other Two is $\frac{\frac{1}{2}p}{n}$

$$- \frac{\frac{1}{6}qq}{np}$$

3° AND

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3^o AND that the Probability of the the third *Life's* surviving the other Two is $\frac{\frac{1}{3}qq}{np}$.

ALL which may be expressed as follows.

1^o TAKE $\frac{1}{2}$ the Square of the mean Complement, and to it add $\frac{1}{6}$ Part of the Square of the shortest; then let the Sum be subtracted from the *Product* of the greater and mean Complements multiplied together.

2^o FROM $\frac{1}{2}$ the Square of the mean Complement, subtract $\frac{1}{6}$ Part of the Square of the shortest.

3^o TAKE $\frac{1}{3}$ Part of the Square of the shortest.

THEN the several Numbers arising from the several Results of these *Operations* will be proportional to the respective Chances which each *Life* taken in Order, viz. the youngest, middlemost, and oldest have, each of them, of surviving the other Two.

THUS supposing three *Lives*, viz. of 40, 50 and 60, the longest, mean, and shortest Complements, will respectively

tively be 46, 36, 26, by help of which Numbers, the three Numbers arising from the *Operations* here prescribed, will be found $895\frac{1}{3}$, $535\frac{1}{3}$, $225\frac{1}{3}$ which are nearly in the Proportion of 13, 8, and 3; and these Numbers will respectively be proportional to the Chances which each *Life* has of surviving the other Two.

PROBLEM XXVI.

T HREE Lives being given, to find the Probability of any Two of them fixed upon outliving the third.

SOLUTION.

1° **T**AKE the *Product* of the longest and mean Complements, from which subtract half the *Product* of the longest and shortest, and from the *Remainder* subtract again half the *Product* of the mean and shortest, then to this last *Remainder* add one third of the *Square* of the shortest Complement.

P

2° FROM

2° FROM half the *Product* of the longest and mean *Complements* subtract one sixth Part of the *Square* of the shortest.

3° FROM half the *Product* of the longest and shortest *Complements* subtract one sixth Part of the *Square* of the shortest.

THEN the Numbers arising from the Results of these several *Operations* will be respectively proportional to the several *Chances* which the youngest and middlemost *Life*, the youngest and oldest, the middlemost and oldest respectively have of outliving the third remaining *Life*.

PROBLEM XXVII.

SUPPOSING Three Persons A, B, C, whereof the First A is in Possession of an Annuity for his Life; the second B to stand next in Reversion exclusive of C, and C to have the Expectation of the Reversion for him and his Heirs for ever, in Case A survives B, to find the Value of the Expectation of C.

SOLU.

SOLUTION.

1° **L**ET us consider what the Value of the Reversion would be to the Expectant *C*, were that Reversion absolute, that is, were there no *Life* intercepted between him and *A*; now by the VIIth Problem it appears, that if from the Value of the Two Lives of *A* and *C*, the *Life* of *A* be subtracted, there will remain the Value of the Reversion. Let us suppose that the *Life* in Possession be worth 11 Years Purchase; it will follow therefore that the Reversion will be worth 9 Years Purchase; for *C* being to have the *Annuity* for him and his Heirs for ever after the *Life* of *A*, the Two Lives of *A* and *C* together are worth 20 Years Purchase, since the *Life* of *C* is to be perpetuated by his Heirs.

2° 9 Years Purchase being the Value of the Reversion to *C*, upon Supposition of an absolute Right, let us consider what Diminution this Value will suffer from the Interposition of the

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Life of *B*, for which let us suppose that the Probability of *A*'s surviving *B* is expressible by the Fraction $\frac{2}{3}$, therefore by the first Principle laid down in the *Doctrine* of Chances, the Value of the Contingent Reversion is $\frac{2}{3}$ of 9, that is 6 Years Purchase.

AND by the same Principle, if there were ever so many *Lives* intervening between the present Possessor *A* and the Expectant *C*, the Value of the Reversion might be determined by previously knowing the Probabilities of Survivorship.



*Dr. HALLER'S TABLE of Observations, exhibiting
the Probabilities of LIFE.*

Age.	Persons.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.
1	1000	8	680	15	628	22	586	29	539	36	481						
2	855	9	670	16	622	23	579	30	531	37	472						
3	798	10	661	17	616	24	573	31	523	38	463						
4	760	11	653	18	610	25	567	32	515	39	454						
5	732	12	646	19	604	26	560	33	507	40	445						
6	710	13	640	20	598	27	553	34	499	41	436						
7	692	14	634	21	592	28	546	35	490	42	427						
Age.	Persons.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.	Age.	Perfo.
43	417	50	346	57	272	64	202	71	131	78	58						
44	407	51	335	58	262	65	192	72	120	79	49						
45	397	52	324	59	252	66	182	73	109	80	41						
46	387	53	313	60	242	67	172	74	98	81	34						
47	377	54	302	61	232	68	162	75	88	82	28						
48	367	55	292	62	222	69	152	76	78	83	23						
49	357	56	282	63	212	70	142	77	68	84	20						

The present Value of an ANNUITY of
One Pound, for any Number of Years not
exceeding 100, Interest at 5 per Cent.

Years	Values.	Years	Values.	Years	Values.	Years	Values.
1	0.9583	26	14.3751	51	18.3389	76	19.5094
2	1.8594	27	14.5430	52	18.4180	77	19.5221
3	2.7232	28	14.8981	53	18.4934	78	19.5350
4	3.5459	29	15.1410	54	18.5651	79	19.5462
5	4.3294	30	15.3724	55	18.6332	80	19.5566
6	5.0756	31	15.5928	56	18.6983	81	19.6156
7	5.7863	32	15.8026	57	18.7605	82	19.6339
8	6.4632	33	16.0025	58	18.8195	83	19.6514
9	7.1078	34	16.1929	59	18.8757	84	19.6680
10	7.7217	35	16.3741	60	18.9292	85	19.6838
11	8.3064	36	16.5468	61	18.9802	86	19.6988
12	8.8632	37	16.7112	62	19.0288	87	19.7132
13	9.3935	38	16.8678	63	19.0750	88	19.7268
14	9.8986	39	17.0170	64	19.1191	89	19.7398
15	10.3796	40	17.1590	65	19.1610	90	19.7522
16	10.8377	41	17.2943	66	19.2010	91	19.7640
17	11.2740	42	17.4232	67	19.2390	92	19.7752
18	11.6895	43	17.5459	68	19.2753	93	19.7859
19	12.0853	44	17.6627	69	19.3098	94	19.7961
20	12.4622	45	17.7740	70	19.3426	95	19.8058
21	12.8211	46	17.8800	71	19.3739	96	19.8151
22	13.1630	47	17.9810	72	19.4037	97	19.8239
23	13.4885	48	18.0771	73	19.4321	98	19.8323
24	13.7986	49	18.1687	74	19.4592	99	19.8402
25	14.0939	50	18.2559	75	19.4849	100	19.8479

MVSEVM
BRITAN
NIQVM S.